Quantum mysteries revisited

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(Received 30 March 1990; accepted for publication 28 April 1990)

A gedanken gadget is described, based on an idea of Greenberger, Horne, and Zeilinger, that provides a more powerful demonstration of quantum nonlocality than Bell's analysis of the Einstein–Podolsky–Rosen experiment.

I. INTRODUCTION

Many years ago I described in these pages a gedanken demonstration,[1] designed to convey the essential character of John Bell's famous analysis[2] of the Einstein–Podolsky–Rosen experiment[3] to somebody unacquainted with the quantum theory. My demonstration consisted of two widely separated detectors, each triggered by one of a pair of particles that originated earlier from a common source. Each detector had a switch that could be set to one of three positions before it was triggered. When triggered it flashed either a red light or a green one. The striking thing about the behavior of this device was that for one long series of runs (in which settings of the two switches always agreed) the resulting data seemed to require, in the absence of spooky actions at a distance, an explanation that was incompatible with the statistical behavior of another long series of runs (in which the two switch settings always disagreed).

Daniel Greenberger, Michael Horne, and Anton Zeilinger[4] recently invented a new version of the EPR experiment which demonstrates the spookiness of quantum mechanics even more dramatically than Bell's analysis of EPR. I describe below a gedanken demonstration in the style of my earlier one, inspired by, but somewhat simpler than, an analysis of the GHZ experiment by Robert Clifton, Michael Redhead, and Jeremy Butterfield.[5] I shall follow the strategy of Ref. 1, first describing the device in entirely nontechnical terms, suitable for an audience of nonscientists but also not inappropriate for an audience of hardened quantum mechanicians to help them shed their quantum mechanical instincts and regain the sense of wonder that such behavior can inspire in the less well trained.

What makes this new device more dramatic than the old one is that the explanation apparently required by the data accumulated in one long series of runs is now refuted not by the *statistics* of the data accumulated in another long series of runs of a different kind, but by the outcome of *one* crucial experiment consisting of a *single* new kind of run.

II. QUANTUM MYSTERIES FOR ANYONE

My new device (pictured in Fig. 1) has *three* widely separated detectors (a complication) but each detector has only *two* switch settings (a simplification). As earlier, a detector, when triggered, flashes red or green. As earlier, all detectors are far apart from the source, there are no connections between the detectors, and no connections between the source and the detectors other than those mediated by a group of particles (now a trio rather than a pair) that originate at the source and fly away, one to each detector.

A run of the experiment consists of setting the switch on each detector to one of its two positions (labeled 1 and 2), pressing a button at the source (to release a trio of particles, one aimed at each detector), and recording the color subsequently flashed at each detector. We only consider the data acquired for four of the eight possible switch settings, those in which the number of detectors set to 1 is odd. (The data for the other four settings are unremarkable and of no relevance for the argument that follows.) We call the detectors A, B, and C, and specify pertinent facts about them by listing the three pieces of information (switch settings or colors flashed) in that order.

If just one detector is set to 1 (and the others to 2), then an odd number of red lights always flash—i.e., either all three detectors flash red, or there is one red flash and two green ones. (All four outcomes—RRR, RGG, GRG, or GGR—are equally likely, but this detail is of no importance.) If all three detectors are set to 1, then an odd number of red lights is never observed to flash—either two of the three flash red or all three flash green.

Let us set aside, for the moment, the 111 case (it will return to haunt us) and consider the cases 122, 212, and 221 in which just one detector is set to 1. Because an odd number of red lights always flash in any of these three cases, whenever the switches are so set we can predict with certainty what any one of the three detectors will do in a

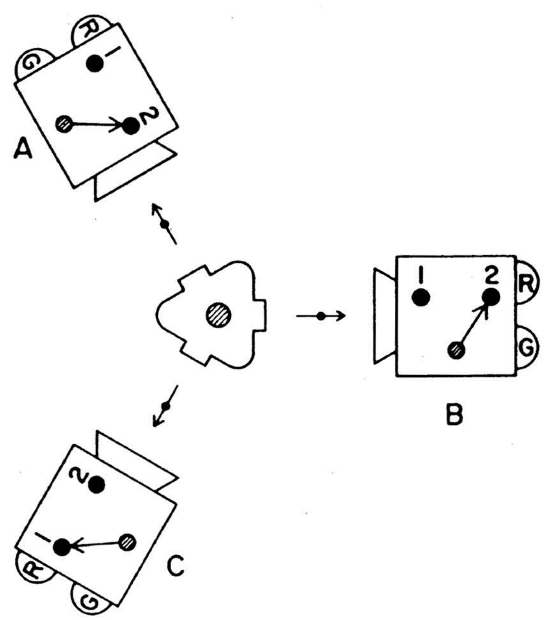

Fig. 1. The three detectors A, B, and C, viewed from above (down the *x* axis). Their switch settings are 221. When the button on the source in the middle is pushed, three particles (shown *en route*) emerge and move in the horizontal plane to the three detectors.

run, merely by noting what happens at the other two. For should the other two flash the same color (RR or GG) then the third will have to flash red, but should the other two flash different colors (RG or GR) then the third will have to flash green.

We now follow Einstein, Podolsky, and Rosen (and the gedanken demonstration of Ref. 1) in drawing an inference that seems well nigh inescapable. Since there are no direct connections between the detectors, their behavior can only be coordinated because all three are triggered by particles that came from a common source. This fact and this fact alone must contain the explanation for why we can learn in advance what color will flash at a given detector, say A, from measurements made far away at B and C. Information telling the detector at A what color to flash in order to maintain the observed consistency with the colors flashed at B and C must somehow be encoded in the particle that triggers A. Since that particle could indeed have been coordinated with the particles that triggered B and C when all three were back at their common source, this explanation seems both inevitable and entirely reasonable.

We can apply this reasoning to any one of the three detectors (by moving it farther from the source so that before it flashes we have the opportunity to observe what colors flash at the other two). We conclude that in each run of the experiment each particle must be carrying to its detector instructions on what color to flash, and that an odd number of the particles must specify red. Thus for a given choice of switch settings (say 122) the particles heading for detectors A, B, and C must respectively be carrying instructions RRR, RGG, GRG, or GGR, but never GRR, RGR, RRG, or GGG. Which of the four allowed groups of instructions they collectively carry is revealed only when the lights flash. All of the above reasoning applies equally well, of course, to 212 and 221 runs.

In the absence of connections between the detectors and the source, a particle has no information about how the switch of its detector will be set until it arrives there. Since in each run any detector might turn out to be either the one set to 1 or one of the ones set to 2, to preserve the perfect record of always having an odd number of red flashes in 122, 212, and 221 runs, it would seem to be essential for each particle to be carrying instructions for how its detector should flash for either of the two possible switch settings it might find upon arrival.

The instructions carried by each particle can be symbolized by a pair of letters: $\genfrac{}{}{0pt}{}{R}{R}$, $\genfrac{}{}{0pt}{}{R}{G}$, $\genfrac{}{}{0pt}{}{G}{R}$, or $\genfrac{}{}{0pt}{}{G}{G}$. The upper letter specifies the color to be flashed if the switch is set to 1, and the lower, the color for setting 2. If a particle is of the type $\genfrac{}{}{0pt}{}{R}{G}$, for example, its detector will flash red if the switch is set to 1 and green if the switch is set to 2.

The totality of flashing instructions carried by the three particles in a given run can be summarized by listing the instructions carried by all three of them. Thus a run in which the instructions carried by the particles were $\begin{smallmatrix}R&G&G\\G&R&R\end{smallmatrix}$ would result in RRR if the switch settings were 122, GGR for 212, and GRG for 221. Since each of the three possible switch settings results in an odd number of red flashes, this is indeed a legal set of instructions. [An example of an illegal set of instructions is $\begin{smallmatrix}R&R&G\\G&R&R\end{smallmatrix}$, for this gives an even number of red flashes (GRR) for the switch setting 212.]

It is not hard to enumerate all the legal instruction sets. First note that three of the six positions in a legal instruction set corresponding to any one of the three choices 122, 212, or 221 for the switch settings, must contain an odd number of R's, since that particular setting might be encountered in any run, and since only odd numbers of red flashes are ever observed. Thus the only possible entries for the positions corresponding to the switch settings 122 are (leaving blank the entries not relevant to those three settings):

$$\begin{matrix}R\!-\,- & R\!-\,- & G\!-\,- & G\!-\,- \\ -RR & -GG & -RG & -GR\end{matrix}\,. \tag{1}$$

We can next count the ways to fill in the blanks in these four forms so as to produce the correct data for switch settings 221. Since each of the four already specifies the color flashed at detector B for setting 2, to ensure that any 221 run produces an odd number of red flashes there are only two choices available for the two unspecified 221 entries, for each of the four forms: RR or GG if the specified entry is R, and RG or GR if the specified entry is G. This raises the number of possible forms to eight, each of which leaves only the entry for setting 1 at detector B unspecified. But that entry is now entirely determined by the entries at settings 2 for detectors A and C (having to be R, if the latter two entries are the same color and G, if they are different). There are thus just eight legal instruction sets.

Out of the 64 possible instruction sets here are the eight legal ones:

$$\begin{matrix}RRR & RGG & GRG & GGR \\ RRR & RGG & GRG & GGR \\ \\ RGG & RRR & GGR & GRG \\ GRR & GGG & RRG & RGR\end{matrix}\,. \tag{2}$$

They are arranged in the same horizontal order as the forms in (1), with the two possibilities for each form placed directly above one another. [It is easy to check explicitly that each instruction set (2) does indeed give an odd number of red flashes when a single detector is set to 1. Since there are only eight legal instruction sets and the eight given in (2) are all legal, you don't actually have to go through the process of filling in the blanks in (1) to know that this has to be the right list.]

Now, finally, we consider the fourth type of run, in which all three detectors are set to 1, and an odd number of red flashes is never observed. The above instruction sets must determine the outcomes of these runs as well. For who is to prevent somebody from flipping the two switches set to 2 over to 1, just before the particles arrive? But an inspection of the upper rows in (2) reveals that *every one of the eight allowed instructions sets results in an odd number of red flashes when all three switches are set to 1.* If the instruction sets existed, then 111 runs would *always* have to produce an odd number of red flashes. But they *never* do, as I remarked in the third paragraph of this section, quite possibly without you strenuously objecting.

Thus a *single* 111 run suffices all by itself to give data inconsistent with the otherwise compelling inference of instruction sets.

This is strikingly more powerful than Bell's theorem for the two-particle EPR experiment. In the version illustrated by the gedanken demonstration of Ref. 1, the existence of instruction sets was apparently forced by the data produced by the two detectors (the colors that flash always agree) when the settings of both switches agreed. It was refuted by the data produced when the two switches had different settings, but that refutation required enough runs to establish a significant difference between the data required by the instruction sets (colors agree at least $33\frac{1}{3}\%$ of the time) and the actual behavior dictated by quantum mechanics (colors agree only 25% of the time). Contrast this with the three-particle device inspired by the GHZ experiment: Instruction sets *require* an odd number of red flashes in *every* 111 run; quantum mechanics *prohibits* an odd number of red flashes in *every* 111 run.

III. QUANTUM SOLUTIONS FOR PHYSICISTS

Here is how the device works. What emerges from the source are three spin-1/2 particles (a, b, and c) in a spin state whose structure is specified below. The particles fly apart to the detectors in the horizontal plane. Define the z direction for each particle to be along its line of flight. The detectors contain Stern–Gerlach magnets which measure the vertical (x) component of the spin when their switch is set to 1 and the horizontal component perpendicular to the line of flight (y) when their switch is set to 2. Red flashes for spin-up, green for spin-down.

Here is a spin state that produces the remarkable Greenberger–Horne–Zeilinger correlations described in Sec. II. Measure angular momentum in units of $\frac{1}{2}\hbar$ so that the spin operators for each particle can be taken to be the Pauli matrices. Consider the three commuting Hermitian operators

$$\sigma_x^a\sigma_y^b\sigma_y^c\,,\quad \sigma_y^a\sigma_x^b\sigma_y^c\,,\quad \sigma_y^a\sigma_y^b\sigma_x^c\,. \tag{3}$$

They commute because all pairs of the six spin operators out of which they are constructed commute, except for those associated with the x and y components of the spin of a single particle, which anticommute. This doesn't cause any trouble, however, because converting the product in one order to the product in the other order always involves an even number of such anticommuting interchanges.

Being commuting and Hermitian, the three operators in (3) can be provided with simultaneous eigenstates. Since the square of each operator is unity, the eigenvalues of each can only be $+1$ or -1. For simplicity we pick the state with all three eigenvalues $+1$, which preserves the symmetry among the particles. The specific form of the state vector is unimportant, but the curious can find it in the Appendix.

Since the components of the spin vectors of different particles commute, we can simultaneously measure the x component for one particle and the y components for the other two. Because the spin state is an eigenstate of all three of the operators (3) with eigenvalue unity, the product of the results of each of the three single spin measurements has to be $+1$, regardless of which particle we pick for the x-spin measurement. Since $+1$ flashes red and -1 flashes green, there must indeed be an even number of green flashes and thus an odd number of reds.

What about the result of three x-spin measurements, declared in Sec. II never to result in an odd number of red flashes? Translating this into spin language tells us that the product of the three results must always be -1. The Hermitian operator corresponding to that product is

$$\sigma_x^a\sigma_x^b\sigma_x^c\,, \tag{4}$$

so for the declaration to be correct, it must be that the eigenstate of the three operators (3) with eigenvalue $+1$ is also an eigenstate of the operator (4) with eigenvalue -1.

This is easily confirmed. Indeed, one readily verifies that the operator (4) is just minus the product of the three operators (3):

$$\sigma_x^a\sigma_x^b\sigma_x^c = -(\sigma_x^a\sigma_y^b\sigma_y^c)(\sigma_y^a\sigma_x^b\sigma_y^c)(\sigma_y^a\sigma_y^b\sigma_x^c)\,. \tag{5}$$

Since we are in an eigenstate with eigenvalue $+1$ of each of the three operators appearing on the right of (5), we are indeed also in an eigenstate of $\sigma_x^a\sigma_x^b\sigma_x^c$ with eigenvalue -1.

Note that the consequence of the EPR reality criterion extracted in Sec. II, if translated into quantum theoretic terminology, would also assert that the state was an eigenstate of the operator (4), *but with the wrong eigenvalue*. In this sense the GHZ experiment provides the strongest possible contradiction between quantum mechanics and the EPR reality criterion.

The crucial minus sign in (5), totally destructive of the possibility of instruction sets, comes from the fact that in working out that identity it is necessary to interchange the anticommuting operators σ_x^b and σ_y^b in order to get rid of all the y components [through $(\sigma_y^i)^2=1$] and be left with a product of three x components. It is only that one instance of uncompensated anticommutation that produces the conclusion so devastating to the hypothesis of instruction sets. This is extremely pleasing, for it is just the fact that the x and y components of the spin of a single particle do not commute, which leads the well-educated quantum mechanician to reject from the start the inference of instruction sets (which have to specify the value of both of these noncommuting observables), making it necessary for me to disguise what was going on in Sec. II in order to prevent so knowledgeable a person from dismissing this article as rubbish before reaching the interesting part.

I know of no other Bellian refutation of Einstein, Podolsky, and Rosen in which the mathematical details of the refutation so closely reflect the broad interpretive doctrines of the quantum theory that EPR tried to challenge. The entries in the instruction sets are precisely the conjectured c-number values for all the σ_x^i and σ_y^i—values that appear to be the only explanation for the remarkable correlations. And the logic of red and green lights in Sec. II precisely parallels the algebraic behavior of the four operators (3) and (4) except for that one devastating anticommutation.

ACKNOWLEDGMENTS

I would like to thank Robert Clifton for sending Jon Jarrett a prepublication copy of Ref. 5 with a suggestion that he might show it to me, and I would like to thank Jarrett for acting on that suggestion and commenting perceptively on an earlier version of this paper. My occasional studies of EPR correlations are supported by a small corner of NSF Grant DMR 89-20979.

APPENDIX

The above argument does not use the form of the spin-state vector that is a simultaneous eigenstate of the three operators (1), but here it is anyway. The state vector is $\Psi=(1/\sqrt{2})(|1,1,1\rangle-|-1,-1,-1\rangle)$, where 1 or -1 specifies spin-up or -down along the appropriate z axis. And the other seven simultaneous eigenstates (corresponding to the other choices of eigenvalue ±1 for each of the three operators) are given by the seven other distinct forms for $(1/\sqrt{2})(|m_1,m_2,m_3\rangle \pm |-m_1,-m_2,-m_3\rangle)$.

This follows immediately from the fact that

$$\sigma_x|\pm 1\rangle = |\mp 1\rangle, \quad i\sigma_y|\pm 1\rangle = \mp|\mp 1\rangle. \tag{A1}$$

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A perspective on teacher preparation in physics and other sciences: The need for special science courses for teachers

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(Received 12 June 1989; accepted for publication 14 November 1989)

This article proceeds from the premise that one of the major reasons for the perceived crisis in science education is the failure of our colleges and universities to provide the type of preparation that precollege teachers need to teach science effectively. The perspective taken is based on many years of teaching physics and physical science to prospective and practicing teachers at all grade levels. The inadequacy of the present system of preparing teachers is examined and an argument is presented for offering special physics courses for teachers. Experience at the University of Washington provides the basis for a discussion of the type of intellectual objectives and instructional methods that should characterize such courses.

I. INTRODUCTION

It is generally accepted that science education in the United States is in serious difficulty. Between the seventh and twelfth grades, the number of students taking science drops by more than 50%.[1] With less than 2 years of science required for graduation by the majority of states,[2] most graduates of American high schools have taken considerably less science than their counterparts in other countries. When achievement is compared, American students do not perform as well as others.[3] If present trends continue, the number of students entering college with both the interest and the preparation to pursue a scientific or technical profession will not be sufficient to meet our national needs.

This article addresses one aspect of the current crisis: the failure of our colleges and universities to provide the type of preparation that precollege teachers need to teach science effectively. The discussion is in terms of physics, but the situation in other sciences is similar.

A. The problem

Over the last 2 decades, the percentage of first-year graduate students in physics who have been educated in this country has been dwindling with respect to the foreign enrollment.[4] This situation is only one consequence of a process that has critical impact beyond the profession: the continual narrowing of the pipeline in physics throughout the period of formal education.[5]

The greatest constriction occurs during the precollege years and is demonstrated by the fact that only about 20% of the students in American high schools study physics.[6,7] Reasons for the steady attrition are complex. Political, social, economic, and intellectual factors all play a role, and it is difficult to separate cause from effect.[8] However, although it cannot be proved, it seems reasonable to assume that one of the most important factors affecting enrollment and retention of students is the shortage of teachers adequately prepared to teach physics.[6,7] According to a recent survey by the American Institute of Physics, about one-third of the teachers with physics assignments have neither majored in the subject nor taught it on a regular basis.[6,7]

The problem of inadequate teacher preparation is not limited to high school, but extends down into middle and elementary school. There, lacking the proper background to teach with enthusiasm and confidence, teachers often transmit to students a dislike of science, especially physical science. With a negative attitude often firmly established by the ninth grade, most students do not voluntarily take physics in high school. Failure to do so decreases the likelihood that students will complete a college course in the