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Wave-function approach to dissipative processes: are there quantum jumps?

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In a recent Letter Dalibard and coworkers have presented an efficient method of computing the development of an open quantum system based on stochastic evolution of the state vector, in which quantum jumps are represented explicitly. Independently of this pragmatic approach, physicists interested in the "quantum measurement problem" have been led to consider continuous stochastic diffusion equations for the state vector associated to any density operator evolution. We underline the remarkable convergence of these two trends in physics, and argue that these recent developments may lead to new results and insights into quantum phenomena.

In Einstein's paper on the A- and B-coefficients [1], he assumed that an individual quantum system like an atom was capable of a transition or jump from one state to another with the absorption or emission of radiant energy. Although Einstein's paper stimulated the development of modern quantum mechanics, such jumps have no formal place in that theory, for which the state vector represents the properties of an ensemble of systems and not an individual system.

Despite the success of the modern theory, many physicists, particularly experimenters, have insisted on treating quantum jumps of individual systems as if they were real, and the state vector as if it represented the behaviour of an individual system, as exemplified by a single run of a laboratory experiment (quantum optics provides many examples, see for instance ref. [2]). And the experimenters' picture has given them valuable physical insights [3], which have sometimes escaped the theoreticians with their relatively elaborate mathematical tools based on density operator evolution. For example Itano and coworkers based their analysis of their "Zeno paradox" experiment on quantum jumps [4], although the same experiment can also be interpreted without jumps by shifting the quantum-classical boundary [5].

Now Dalibard, Castin and Molmer [6], and also Carmichael [7] and Teich and Mahler [8] have come up with an efficient method of computing the development of an open system based on stochastic evolution of the state vector, in which quantum jumps are represented explicitly. In addition to the advantages listed in the above references, this method, contrary to some others, as explained in ref. [9], has no problem with the uncertainty relations.

Independently of these developments, Bohm, Bub, Pearle [10], Gisin [11], Ghirardi, Rimini and Weber, Diósi, and Bell [12] have proposed alternative quantum theories in which the state vector represents an individual system and follows a stochastic dynamics.

In particular Diósi, Gisin and Pearle have obtained continuous stochastic diffusion equations for the state vector from any density operator evolution equation. Percival [13] has provided a natural symmetry condition under which this state vector equation is unique, and has suggested that the equations should be used as a practical tool. Given Lindblad's [14] expression for the equation of motion for the density operator

$$\dot{\rho} = -\mathrm{i}[H,\rho] + \sum_n \left(2L_n\rho L_n^\dagger - \{L_n^\dagger L_n, \rho\}\right), \tag{1}$$

the differential form of the stochastic equation of motion for the state vector $|\psi\rangle$ is

$$\mathrm{d}|\psi\rangle = \left(-\mathrm{i}H|\psi\rangle - \sum_n L_n^\dagger L_n|\psi\rangle\right)\mathrm{d}t + \sum_n \left(\mathrm{d}\xi_n + 2\langle\psi|L_n^\dagger|\psi\rangle\,\mathrm{d}t\right)L_n|\psi\rangle, \tag{2}$$

where $\mathrm{d}\xi_n$ represents the Itô form of the complex normalized Wiener process that satisfies

$$\mathrm{Re}(\mathrm{d}\xi_n)\,\mathrm{Re}(\mathrm{d}\xi_m) = \mathrm{Im}(\mathrm{d}\xi_n)\,\mathrm{Im}(\mathrm{d}\xi_m) = \delta_{nm}\,\mathrm{d}t,$$
$$\mathrm{Re}(\mathrm{d}\xi_n)\,\mathrm{Im}(\mathrm{d}\xi_m) = 0. \tag{3}$$

The theory is given in ref. [11,15]. Equation (2) can be used, like those of Dalibard et al., to provide an efficient Monte Carlo solution of problems that are normally formulated in terms of the time evolution of the density operator, and applications are already in progress.

This method has the following advantages over the methods of Dalibard and coworkers. Equation (2) is derived explicitly and uniquely from any Markovian density operator evolution equation and its solutions are continuous in time. Since it uses the well developed Itô stochastic calculus, all the existing tools [16] for numerical integrations of such stochastic equations can be used. The uniqueness of the diffusion equation provides a one-to-one relation between a distribution of pure states at an initial time, and the distribution at later times.

Figure 1 illustrates an application of our method to the non-linear absorber:

$$\dot{\rho} = 0.1[(a^\dagger - a), \rho] + 2a^2\rho a^{\dagger 2} - \{a^{\dagger 2}a^2, \rho\}. \tag{4}$$

This example has also been treated with the positive P-representation (an extension of the Wigner function), but it suffers from runaway problems [17]. The full line in fig. 1 is the direct solution of the density operator equation. The result based on eq. (2),

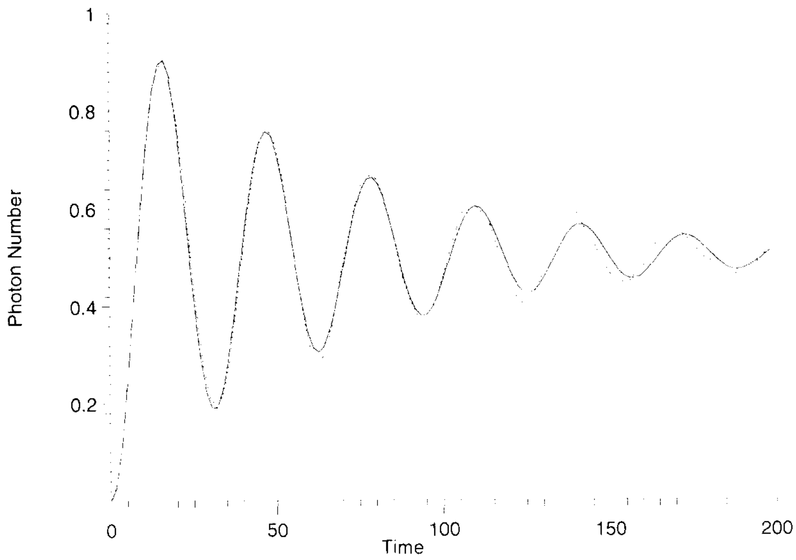

Fig. 1. Density operator (full line) and stochastic diffusion (dotted line) methods compared for the nonlinear absorber, eq. (4). The used time increment is $\mathrm{d}t=0.02$ and 100 samples are used for approximating the average.

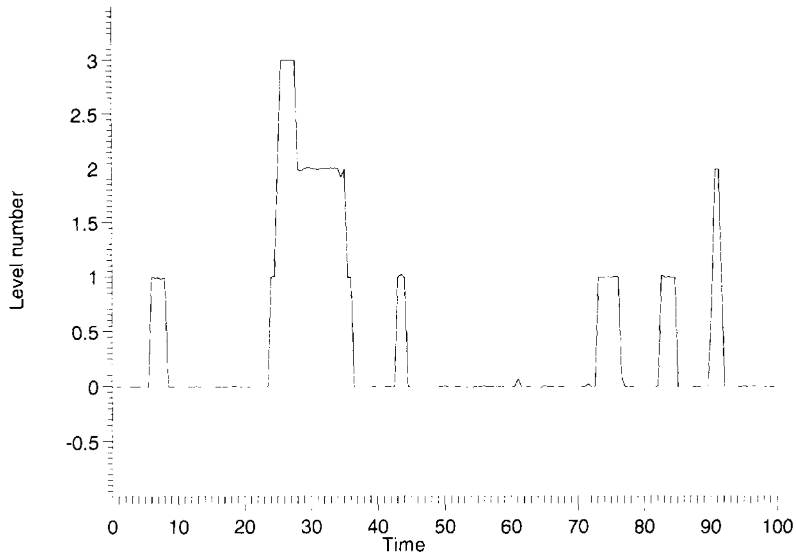

Fig. 2. A sample trajectory of the stochastic process associated to eq. (1) for a spin 3/2 (i.e. a four-level system) with a weak pump ($h=\mathrm{i}(S_+-S_-)$), a weak spontaneous decay ($L_1=0.2S_-$) and a measurement like interaction ($L_2=5S_z$). Note that the solutions of eq. (1) have many fine spikes that are not seen on the figure because of the time discretization.

the dotted line, agrees for all times, without large fluctuations, contrary to the positive P-representation. Other applications are investigated in ref. [18]. In particular fast transitions between quasi-stable states appear naturally, as well as specific interactions with the environment leading to measurement like reductions of the state vector, see fig. 2.

There is thus a remarkable convergence of two trends in physics that have previously been quite distinct: The quantum measurement "problem" as considered by physicists worried by the foundations of quantum physics, and the quantum measurement process as treated pragmatically by experimenters looking for intuitive pictures and rules for computation.

It can now be seen that the stochastic reduction picture of quantum mechanics provides both insight and practical tools for the solution of physical problems.

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