Assessment of Everett's "Relative State" Formulation of Quantum Theory

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THE preceding paper puts the principles of quantum mechanics in a new form.[1] Observations are treated as a special case of normal interactions that occur within a system, not as a new and different kind of process that takes place from without. The conventional mathematical formulation with its well-known postulates about probabilities of observations is derived as a *consequence* of the new or "*meta*" quantum mechanics. Both formulations apply as well to complex systems as to simple ones, and as well to particles as to fields. Both supply mathematical models for the physical world. In the new or "relative state" formalism this model associates with an isolated system a state function that obeys a linear wave equation. The theory deals with the totality of all the possible ways in which this state function can be decomposed into the sum of products of state functions for subsystems of the overall system—and nothing more. For example, in a system endowed with four degrees of freedom x_1,x_2,x_3,x_4, and a time coordinate, t, the general state can be written $\psi(x_1,x_2,x_3,x_4,t)$. However, there is *no* way in which ψ defines any unique state for any subsystem (subset of x_1,x_2,x_3,x_4). The subsystem consisting of x_1 and x_3, say, *cannot* be assigned a state $u(x_1,x_3,t)$ independent of the state assigned to the subsystem x_2 and x_4. In other words, there is ordinarily *no* choice of f or u which will allow ψ to be written in the form $\psi=u(x_1,x_3,t)f(x_2,x_4,t)$. The most that can be done is to associate a *relative* state to the subsystem, $u_{\mathrm{rel}}(x_1,x_3,t)$, relative to some *specified* state $f(x_2,x_4,t)$ for the remainder of the system. The method of assigning relative states $u_{\mathrm{rel}}(x_1,x_3,t)$ in one subsystem to specific states $f(x_2,x_4,t)$ for the remainder, permits one to decompose ψ into a superposition of products, each consisting of one member of an orthonormal set for one subsystem and its corresponding relative state in the other subsystem:

$$\psi=\sum_i a_i f_i(x_2,x_4,t)u_{\mathrm{rel}\,f_i}(x_1,x_3,t), \qquad (1)$$

where $\{f\}$ is an orthonormal set. According as the functions f_n constitute one or another family of orthonormal functions, the relative state functions $u_{\mathrm{rel}\,f_n}$ have one or another dependence upon the variables of the remaining subsystem.

Another way of phrasing this unique association of relative state in one subsystem to states in the remainder is to say that the states are correlated. The totality of these correlations which can arise from all possible decompositions into states and relative states is all that can be read out of the mathematical model.

The model has a place for observations only insofar as they take place within the isolated system. The theory of observation becomes a special case of the theory of correlations between subsystems.

How does this mathematical model for nature relate to the present conceptual scheme of physics? Our conclusions can be stated very briefly: (1) The conceptual scheme of "relative state" quantum mechanics is completely different from the conceptual scheme of the conventional "external observation" form of quantum mechanics and (2) The conclusions from the new treatment correspond completely in familiar cases to the conclusions from the usual analysis. The rest of this note seeks to stress this *correspondence in conclusions* but also this *complete difference in concept.*

The "external observation" formulation of quantum mechanics has the great merit that it is dualistic. It associates a state function with the system under study—as for example a particle—but not with the *ultimate* observing equipment. The system under study can be enlarged to include the original object as a subsystem and also a piece of observing equipment—such as a Geiger counter—as another subsystem. At the same time the number of variables in the state function has to be enlarged accordingly. However, the *ultimate* observing equipment still lies outside the system that is treated by a wave equation. As Bohr[2] so clearly emphasizes, we always interpret the wave amplitude by way of observations of a classical character made from outside the quantum system. The conventional formalism admits no other way of interpreting the wave amplitude; it is logically self-consistent; and it rightly rules out any classical description of the internal dynamics of the system. With the help of the principle of complementarity the "external observation" formulation nevertheless keeps all it consistently can of classical concepts. Without this possibility of classical measuring equipment the mathematical machinery of quantum mechanics would seem at first sight to admit no correlation with the physical world.

Instead of founding quantum mechanics upon classical physics, the "relative state" formulation uses a completely different kind of model for physics. This new model has a character all of its own; is conceptually

[1] Hugh Everett, III, Revs. Modern Phys. **29**, 454 (1957).

[2] Chapter by Niels Bohr in *Albert Einstein, Philosopher-Scientist*, edited by P. A. Schilpp (The Library of Living Philosophers, Inc., Evanston, Illinois, 1949).

self-contained; defines its own possibilities for interpretation; and does not require for its formulation any reference to classical concepts. It is difficult to make clear how decisively the "relative state" formulation drops classical concepts. One's initial unhappiness at this step can be matched but few times in history[3]: when Newton described gravity by anything so preposterous as action at a distance; when Maxwell described anything as natural as action at a distance in terms as unnatural as field theory; when Einstein denied a privileged character to any coordinate system, and the whole foundations of physical measurement at first sight seemed to collapse. How can one consider seriously a model for nature that follows neither the Newtonian scheme, in which coordinates are functions of time, nor the "external observation" description, where probabilities are ascribed to the possible outcomes of a measurement? Merely to analyze the alternative decompositions of a state function, as in (1), without saying what the decomposition means or how to interpret it, is apparently to define a theoretical structure almost as poorly as possible! Nothing quite comparable can be cited from the rest of physics except the principle in general relativity that all regular coordinate systems are equally justified. As in general relativity, so in the relative-state formulation of quantum mechanics the analysis of observation is the key to the physical interpretation.

Observations are not made from outside the system by some super-observer. There is no observer on hand to use the conventional "external observation" theory. Instead, the whole of the observer apparatus is treated in the mathematical model as part of an isolated system. All that the model will say or ever can say about observers is contained in the interrelations of eigenfunctions for the object part of this isolated system and relative state functions of the remaining part of the system. Every attempt to ascribe probabilities to observables is as out of place in the relative state formalism as it would be in any kind of quantum physics to ascribe coordinate and momentum to a particle at the same time. The word "probability" implies the notion of observation from outside with equipment that will be described typically in classical terms. Neither these classical terms, nor observation from outside, nor a priori probability considerations come into the *foundations* of the relative state form of quantum theory.

So much for the conceptual differences between the new and old formulations. Now for their correspondence. The preceding paper shows that this correspondence is detailed and close. The tracing out of the correspondence demands that the system include something that can be called an observing subsystem. This subsystem can be as simple as a particle which is to collide with a particle that is under study. In this case the correspondence occurs at a primitive level between the relative state formalism where the system consists of two particles, and the external observation theory where the system consists of only one particle. The correlations between the eigenfunctions of the object particle and the relative state functions of the observer particle in the one scheme are closely related in the other scheme to the familiar statements about the relative probabilities for various possible outcomes of a measurement on the object particle.

A more detailed correspondence can be traced between the two forms of quantum theory when the observing system is sufficiently complex to have what can be described as memory states. In this case one can see the complementary aspects of the usual external observation theory coming into evidence in another way in the relative state theory. They are expressed in terms of limitations on the degree of correlation between the memory states for successive observations on a system of the same quantity, when there has been an intervening observation of a noncommuting quantity. In this sense one has in the relative state formalism for the first time the possibility of a closed mathematical model for complementarity.

In physics it is not enough for a single observer or apparatus to make measurements. Different pieces of equipment that make the same type of measurement on the same object system must show a pattern of consistency if the concept of measurement is to make sense. Does not such consistency demand the external observation formulation of quantum theory? There the results of the measurements can be spelled out in classical language. Is not such "language" a prerequisite for comparing the measurements made by different observing systems?

The analysis of multiple observers in the preceding paper by the theory of relative states indicates that the necessary consistency between measurements is already obtained without going to the external observer formulation. To describe this situation one can use if he will the words "communication in clear terms always demands classical concepts." However, the kind of physics that goes on does not adjust itself to the available terminology; the terminology has to adjust itself in accordance with the kind of physics that goes on. In brief, the problem of multiple observers solves itself within the theory of relative states, not by adding the conventional theory of measurement to that theory.

It would be too much to hope that this brief survey should put the relative state formulation of quantum theory into completely clear focus. One can at any rate end by saying what it does not do. It does not seek to supplant the conventional external observer formalism, but to give a new and independent foundation for that formalism. It does not introduce the idea of a super-observer; it rejects that concept from the start. It does not supply a prescription to say what is the correct

[3] See, for example, Philipp Frank's *Modern Science and Its Philosophy* (George Braziller, New York, 1955), Chap. 12.

functional form of the Hamiltonian of any given system. Neither does it supply any prediction as to the functional dependence of the over-all state function of the isolated system upon the variables of the system. But neither does the classical universe of Laplace supply any prescription for the original positions and velocities of all the particles whose future behavior Laplace stood ready to predict. In other words, the relative state theory does not pretend to answer all the questions of physics. The concept of relative state does demand a totally new view of the foundational character of physics. No escape seems possible from this relative state formulation if one wants to have a complete mathematical model for the quantum mechanics that is internal to an isolated system. Apart from Everett's concept of relative states, no self-consistent system of ideas is at hand to explain what one shall mean by quantizing[4] a closed system like the universe of general relativity.

[4] C. W. Misner, Revs. Modern Phys. 29, 497 (1957).

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Interaction of Neutrinos and Gravitational Fields

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1. INTRODUCTION; GRAVITATION THE ONLY FORCE IN WHICH NEUTRINOS ARE SUBJECT TO SIMPLE ANALYSIS

KNOWLEDGE of neutrinos to date is confined mainly to emission and absorption processes; that is, to the domain of elementary particle transformations. For comparison, imagine that one knew about electrons only the rate at which they are produced in beta decay, or absorbed in K-electron absorption processes, but knew nothing about the motion of electrons in electric and magnetic fields, nothing about the binding of electrons in atoms or the existence of spin-orbit coupling and very little about the stress energy tensor of the electron. What can one do to learn some fraction as much about neutrinos as one knows today about electrons?

The neutrino does not respond directly to electric or magnetic fields. Therefore, if one wishes to influence its orbit by forces subject to simple analysis one has to make use of gravitational fields. In other words, one has to consider the physics of a neutrino in a curved metric.

For this task the only available tools of analysis are theoretical. We accept the recently clarified[1] and dramatically tested[2,3] neutrino theory. We see no motive to change the theory. Instead we recall in Sec. 2 the clearly defined extension of the Dirac equation to the curved space that represents the most general gravitational field. In Sec. 3, we specialize to the neutrino with its zero mass and to the class of solutions with right-handed polarization that are demanded by the recently gained knowledge.[1–3] Section 4 separates out the radial wave equation for the motion of a neutrino in a centrally symmetric gravitational field, and identifies one term in this equation with a spin-orbit coupling. Section 5 compares and contrasts the energy level spectrum in the case of spherical symmetry for (1) an electron in an electrostatic field, (2) an electron in a gravitational field, (3) a photon in a gravitational field, and (4) a neutrino in a gravitational field. Section 6 recalls the statistical mechanics of an ensemble of neutrinos. Section 7 discusses some neutrino pair creation processes that do not depend upon beta interactions for their existence. Section 8 deals with the contribution of neutrinos to the stress energy tensor, Sec. 9 deals with the gravitational interaction of two neutrinos traveling parallel or antiparallel to each other; and Sec. 10 with the contribution to the stress energy tensor due to a neutrino in a bound orbit. Finally, Sec. 11 examines by way of illustration an object where both the creation of gravitational fields by neutrinos, and the response of neutrinos to gravitational fields come into play: a geon or entity constituted entirely of neutrinos and held together by their mutual gravitational attractions.

[1] T. D. Lee and C. N. Yang, Phys. Rev. 104, 254 (1956); 105, 1119(L) (1957).
[2] Wu, Ambler, Hayward, Hoppes, and Hudson, Phys. Rev. 105, 1413(L) (1957).
[3] Garwin, Lederman, and Weinrich, Phys. Rev. 105, 1415(L) (1957).

2. MATHEMATICS OF SPIN IN CURVED SPACE

Spinor fields have been treated in general relativity by many authors[4] and from three principal points of view (Table I). The three formalisms are in principle equivalent and must therefore in any actual problem give identical results for such well-defined quantities as

[4] See W. L. Bade and H. Jehle, Revs. Modern Phys. 25, 714 (1953) for a general review of the literature. To their list of principal references one should add M. Riesz, Lund Univ. Math. Sem. Band 12 (1954); F. J. Belinfante, Physica 7, 305 (1940).