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Hidden Variables and Quantum Uncertainty
(Written Symposium, 1st Issue)

Variables cachées et indéterminisme quantique
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Verborgene Parameter und Quanten-Unbestimmtheit
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De quoi s'agit-il ? - Introduction élémentaire

Début 19e siècle - Fresnel prouve, par des expériences considérées comme irréfutable, le caractère ondulatoire de la lumière.

1905 Einstein suggère d'expliquer l'effet photoélectrique par une théorie corpusculaire de la lumière, faisant appel aux quanta récemment découverts par Planck.

1924 Pour expliquer les nombres entiers (nombres quantiques) qui apparaissaient dans la théorie des quanta et en se basant sur la mécanique de Hamilton-Jacobi, de Broglie propose inversément d'associer une onde aux corpuscules matériels tels que l'électron.

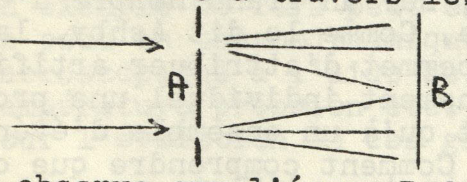
On a donc, dans ces deux domaines, coexistence d'une théorie ondulatoire et d'une théorie corpusculaire. Le problème se pose de les concilier.

1926 Born interprète l'amplitude de l'onde (ou plutôt le carré de son module) comme une densité de probabilité de présence du corpuscule, interprétation universellement acceptée aujourd'hui. Bohr et Heisenberg proposent d'introduire la notion de complémentarité: suivant l'expérience qu'on se propose de faire, la réalité physique apparaît tantôt sous l'aspect corpusculaire, tantôt sous l'aspect ondulatoire. Les expériences faisant apparaître l'un ou l'autre aspect s'excluant mutuellement, il n'y a jamais contradiction expérimentale. Mais notre conception de la réalité physique doit être une synthèse de ces deux aspects contradictoires et pourtant complémentaires.

1927 Au congrès Solvay, grande discussion entre les grands physiciens théoriciens. Einstein conteste l'interprétation de Bohr et Heisenberg (Ecole de Copenhague). De Broglie propose une théorie dite de l'onde pilote où il admet que la trajectoire du corpuscule est déterminée par l'onde telle qu'elle est à l'endroit ou il se trouve (théorie de l'onde pilote).

Car il faut bien dire que l'interprétation probabiliste de Born ne résoud pas tous les problèmes.

Prenons par exemple l'expérience d'Young:
On perce deux trous dans un écran A et on examine ce qui se passe sur l'écran B lorsqu'on envoie de la lumière à travers les trous



On observe sur l'écran B des franges d'interférence qui s'expliquent très bien dans une théorie ondulatoire.

Par contre, dans une théorie corpusculaire, on a de grosses difficultés. Car la figure d'interférence qu'on obtient sur l'écran B n'est pas simplement la superposition de ce qu'on obtiendrait avec chaque trou séparément. A certains endroits, il y a moins de lumière lorsque les deux trous sont ouverts que lorsque un seul l'est. C'est d'ailleurs des expériences de ce type qui avaient permis à Fresnel, d'imposer sa théorie ondulatoire.

Supposons maintenant qu'on envoie une lumière très faible, de telle sorte qu'il ne tombe sur le dispositif qu'un grain de lumière (photon) de temps à autre. Ce photon passera soit par un trou, soit par l'autre.

La trajectoire du photon qui passe par un trou doit donc être modifiée par le fait que le second trou soit ouvert ou non, même s'il ne passe pas de photon par ce second trou.

Or que peut-il passer par ce second trou? On a une réponse toute prête : l'onde associée au corpuscule.

Mais si cette onde ne représente qu'une probabilité de présence, comment comprendre qu'elle influence physiquement la trajectoire du corpuscule? Car une probabilité, sur un événement individuel, ne représente rien de réel: ou bien un photon est dans un certain domaine ou bien il n'y est pas. La probabilité ne prend un sens que statistiquement, sur un grand nombre d'événements de même type. Comme le dit Ashby, la notion probabilité permet d'attribuer artificiellement à un événement individuel une propriété qui n'appartient qu'à un ensemble d'événements de même type. Comment comprendre que cet ensemble d'événements, qui ne sont pas concrètement présents au moment du passage du photon, influencent son mouvement?

On comprend donc que L. de Broglie ait été conduit à formuler une théorie dite de la double solution, où il suppose que l'équation de Schrödinger admet deux solutions: l'une étant l'onde ψ donnant la probabilité de présence, l'autre étant une onde physique u qui pourrait influencer la trajectoire du corpuscule de telle sorte que, statistiquement, la probabilité de présence du corpuscule redonne celle prévue par l'onde ψ. (Plus exactement, l'onde u serait une solution non linéaire comportant des singularités qui représenteraient les corpuscules.) Mais, devant les difficultés mathématiques rencontrées, il s'était rabattu sur la théorie plus simple de l'onde pilote.

La discussion entamée au congrès Solvay ne s'est jamais complètement tue. La majorité des physiciens s'est ralliée à l'interprétation de Copenhague, qui refuse de se poser des questions quant au déroulement d'un événement individuel dans l'espace-temps, ne voulant s'intéresser qu'à ce qu'on peut effectivement mesurer.

Mais quelques physiciens, et non des moindres (Einstein, Schrödinger, De Broglie, Bohm), tout en admettant les succès expérimentaux de la mécanique quantique, sont restés d'avis qu'il faudrait obtenir une description ayant un véritable caractère de réalité, c'est-à-dire ne dépendant pas de ce que l'observateur a arbitrairement décidé de mesurer.

Un exemple me fera peut-être mieux comprendre: on peut en thermodynamique, décrire un système macroscopique à l'aide de grandeurs globales: température, pression etc. et énoncer des lois pour l'évolution de ces grandeurs.

Boltzmann et Gibbs ont réussi, à partir d'un modèle de molécules obéissant aux lois de la mécanique, à retrouver ces grandeurs macroscopiques et les lois qui les régissent.

Du point de vue de la thermodynamique macroscopique, les positions et les vitesses des molécules individuelles sont des variables cachées, non prévisibles. Mais on peut faire des hypothèses raisonnables sur ces variables cachées et retrouver la thermodynamique, ses grandeurs et ses lois.

En ce sens, la thermodynamique macroscopique (ou "phénoménologique") n'est pas une description complète; elle ne traite les molécules que par des grandeurs statistiques, globales, moyennes; elle ne tient pas compte des mouvements et des écarts individuels et se montre incapable de prédire des phénomènes tels que les fluctuations.

De même, d'après Einstein et d'autres, la mécanique quantique ne serait pas une description complète. Il faudrait inventer une théorie faisant certaines hypothèses sur des variables cachées, inacessibles à l'expérience, (telles que vitesse et position, trajectoire des corpuscules), théorie qui sous-tendrait la mécanique quantique comme la mécanique statistique sous-tend la thermadynamique phénoménologique. Et c'est notre ignorance de la valeur de certains paramètres qui nous obligerait à nous rabattre sur une description statistique. L'indéterminisme ne serait pas dans les choses, il proviendrait de notre ignorance.

Les faits nouveaux sont les suivants:

Bell a pu montrer, en 1965, que, moyennant certaines hypothèses raisonnables, les théories à variables cachées devaient satisfaire à une certaine inégalité, inégalité qui permettrait de les différencier des prévisions de la mécanique quantique.

Une expérience a été proposée par Shimony, Horne, Holt et Clauser puis exécutée par Freedman et Clauser. Elle a donné un résultat donnant nettement raison à la mécanique quantique et excluant au moins un certain type de théories à variables cachées.

La discussion est ouverte pour évaluer la portée exacte de ces résultats et les conséquences qu'on doit en tirer.

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LOCAL HIDDEN-VARIABLE THEORIES

We shall initiate the colloquium on the status of local hidden-variable theories by commenting on four topics. First, we shall present a version of the argument of Einstein, Podolsky, and Rosen, concluding that quantum mechanics needs to be completed by a local hidden-variable theory. Next, we shall demonstrate that a generalized Bell's inequality holds for every local hidden-variable theory and shall show that there are situations for which the quantum mechanical predictions violate the inequality. Then we shall sketch an experiment based upon this discrepancy, proposed by Clauser, Holt, and ourselves, and carried out by Freedman and Clauser, with results favoring quantum mechanics. Finally, we shall discuss the possibility of new experiments dispensing with certain assumptions, which a dedicated advocate of hidden-variable theories might challenge, made in the foregoing experiment.

The argument of Einstein, Podolsky, and Rosen[1] concerns a system composed of two spatially separated components, such that the quantum state of the composite system cannot be represented as a product of quantum states of the components. An example is a pair of photons propagating respectively in the $\hat{z}$ and $-\hat{z}$ directions, in the state of polarization

$$\Psi = \frac{1}{\sqrt{2}}\left[\begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}\right], \quad (1)$$

where $\begin{pmatrix}1\\0\end{pmatrix}$ designates linear polarization along the x-axis and $\begin{pmatrix}0\\1\end{pmatrix}$ designates linear polarization along the y-axis. We are interested in two characteristics of the state Ψ. The first is the correlation of the polarizations of the two photons: if one determines that the first photon is polarized along the x-axis, or along the y-axis, one can predict with certainty that the second photon would exhibit a polarization along the same axis. The second characteristic is that Ψ can be written equally well in the form

(1')
$$\Psi = \frac{1}{\sqrt{2}}\left[\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix}\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix} + \begin{pmatrix}-\sin\theta\\ \cos\theta\end{pmatrix}\begin{pmatrix}-\sin\theta\\ \cos\theta\end{pmatrix}\right]$$

But $\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix}$ designates polarization along the axis x', and $\begin{pmatrix}-\sin\theta\\ \cos\theta\end{pmatrix}$ designates polarization along the axis y', where the primed axes are obtained by an arbitrary rotation θ from the original axes. Consequently, Ψ is also a state in which one finds correlations of the polarizations of the two photons relative to the primed axes. The argument of Einstein, Podolsky, and Rosen now proceeds in the following manner. The experimenter can choose to observe the linear polarization of the first photon relative to the unprimed axes or relatively to the primed axes. According to his choice, the polarization of the second photon will be well-determined relative to either the former or the latter set of axes, in view of the correlations implicit in Ψ. In neither case is the second photon subjected to any perturbation by the experimenter, if one supposes the non-occurrence of action-at-a-distance . Consequently, each attribute of the second photon which would be well-determined if the experimenter made an appropriate choice, must be well-determined in advance, without regard for the experiment performed upon the first photon. Thus, we have succeeded in describing the second photon in a more complete manner than that of quantum mechanics, because (except in a trivial case) there is no quantum state in which the polarization can be well-determined relative to two systems of axes.

One often hears the objection that the argument of Einstein, Podolsky, and Rosen passes illegitimately from the conditional to the indicative, not taking account of the fact the choices of the experimenter are mutually exclusive. One can reply, by reference to logical considerations, that a mathematical theorem can be demonstrated on the basis of a disjunction $p_1 \vee p_2$ without knowing the truth or the falsity of each component of the disjunction. (We must admit, however, that the intuitionist school of mathematics does not accept the legitimacy of such a demonstration - a fact which indicates the possibility of a liaison beween intuitionist logic and quantum logic.) One can also reply, by reference to psychological considerations, that children observed in Professor Piaget's laboratory exhibit a mastery of conditional reasoning at the age of 12 years,[2] and that mature physicists and philosophers ought to reason with equal finesse. Actually, however, what is at stake is more a matter of metaphysics than of logic or psychology. The conditional reasoning of children is intimately bound up with theirrealistic conception of objects, that is to say, that objects possess their attributes intrinsically, without consideration of whether they are observed or not. The coherence and depth of Bohr's criticism[3] of the argument of Einstein, Podolsky, and Rosen is due to his linkage of the objection to conditional reasoning with his disavowal of a realistic conception of microphysical objects. (We do not have space in the present paper to discuss the Bohr-Einstein controversy. We shall remark only that Bohr's philosophical critique seems to us far from decisive, but that the outcome of the experiment reported below seriously weakens Einstein's position.)

While concluding that the quantum mechanical description of a physical object is not complete, Einstein, Podolsky, and Rosen did not indicate the form of a more complete description. One can reasonably conjecture that they considered an object at each moment to be in a definite state λ, analogous to a point in the phase-space of a classical mechanical system. The result of the measurement of an arbitrary observable O of the object would be a welldefined value $O(\lambda)$ (though one can also consider hidden-variable theories in which λ determines only a probability distribution of possible outcomes of measuring O, and the generalized Bell's inequality can also be demonstrated for such theories[4]). The hidden variables, properly speaking, are those components of the state λ which are not contained in the quantum state. In order to make statistical predictions on an ensemble of similar objects, one must suppose that a probability distribution μ is given on the space Ω of definite states - that is to say, a non-negative and additive function of measurable regions of Ω such that the whole space is assigned the value 1. Since Einstein, Podolsky, and Rosen expressed no doubts that the statistical predictions of quantum mechanics are correct,[5] one can suppose that they envisaged a correspondance between quantum states Ψ and distributions μ_Ψ such that the expectation value of any observable O calculated with respect to μ_Ψ would equal the quantum mechanical expectation value:

$$\int_\Omega O(\lambda)\, d\mu_\Psi = \langle\Psi|O|\Psi\rangle \quad . \qquad (2)$$

The program which we have just now attributed to Einstein, Podolsky, and Rosen encounters some serious difficulties because of the non-classical structure of the family of quantum mechanical observables. These difficulties are posed by some well-known theorems of von Neumann, Jauch and Piron, and Gleason. Without giving any details, we shall assert that these difficulties are not insurmountable, and we refer to a paper of J.S. Bell[6] for a deep analysis of them.

More serious, in our opinion, is the difficulty posed by the following discovery of Bell himself[7]: that any local theory of hidden variables violates the condition (1).

In order to understand the word "local" in the sense of Bell, one must consider a system consisting of two spatially separated components. Let $\{A_a\}$ be a family of observables of the first component, parametrized by a; and let $\{B_b\}$ be a family of observables of the second component, parametrized by b. Among the observables of the composite system will be found products of the form $A_a \cdot B_b$, which in a hidden-variable theory of the type attributed to Einstein, Podolsky, and Rosen are functions of λ. The condition of locality in the sense of Bell is the following proposition, assumed to hold for all a and b:

$$(A_a \cdot B_b)(\lambda) = A_a(\lambda) \cdot B_b(\lambda). \qquad (3)$$

For example, the system may be a photon pair of the sort considered above, and A_a may be the observable which has the value 1 if the first photon is polarized along an axis in the xy-plane making an angle a with the x-axis, while it has the value -1 if the first photon is polarized along an axis making an angle a with the y-axis; and B_b has an analogous interpretation. There will be a correlation between A_a and B_b, because both depend upon the definite state λ of the photon pair. The essential point, however, is that A_a does not depend upon b, nor does B_b depend upon a. Bell's conception of locality is clearly in accordance with common sense, which affirms that the outcome of an experiment performed upon one component of a composite system should not depend upon the choice of an observable of the other component, assumed to be spatially separated from the first component.

We shall now demontrate[8] the generalized Bell's inequality, from which the falsity of (2) will be seen to follow. Let us limit our attention to observables, such as those of the previous paragraph, which can take on only the values 1 and -1, and let us consider only two members A_a and $A_{a'}$ of the first family and two members B_b and $B_{b'}$ of the second family. The space Ω can then be decomposed into 16 mutually exclusive and exhaustive subspaces (++;++), (++;+-), ... , (--;--). In this notation, the first argument place is + or - according as A_a has the value 1 or -1, the second argument place is similarly associated with $A_{a'}$, and the 3rd and 4th argument places are similarly associated with B_b and $B_{b'}$. For example, (+-;-+) is the subspace in which A_a, $A_{a'}$, B_b and $B_{b'}$ have respectively the values 1, -1, -1, 1.

In general one cannot measure A_a and $A_{a'}$ simultaneously, but this does not prevent us from thinking conditionally: <u>if</u> A_a were measured a certain value would be found, and <u>if</u> $A_{a'}$ were measured a certain value would be found. On the other hand, because of the spatial separation of the two components an arbitrary member of the first family can be measured simultaneously with an arbitrary member of the second family. When a distribution μ is given on Ω, the probabilities μ (++;++) etc. are well-defined and non-negative, and their sum is 1. Let us now define the correlation function

$$E(a,b)_{lhv} = \int_{\Omega} A_a(\lambda)B_b(\lambda)d\mu \,, \qquad (4)$$

with similar definitions for $E(a,b')_{lhv}$, $E(a',b)_{lhv}$, and $E(a',b')_{lhv}$. (The subscript "lhv" designates "local hidden-variable".)

Intuitively, E(a,b) measures the correlation of the observable A_a with the observable B_b. Indeed, if A_a and B_b agree in sign for every value of λ, then clearly E(a,b) = 1, while if they disagree in sign for each walue of λ, then E(a,b) = -1; partial agreement in sign leads to a value of E(a,b) between -1 and 1.

Evidently, if we note the value, either 1 or -1, of $A_a(\lambda)B_b(\lambda)$, within each of the subspaces, Eq. (4) becomes

$$E(a,b)_{lhv} = \mu(++;++) + \mu(++;+-) - \mu(++;-+) - \mu(++;--) + \mu(+-;++) + \mu(+-;+-) - \mu(+-;-+) - \mu(+-;--) - \mu(-+;++) - \mu(-+;+-) + \mu(-+;-+) + \mu(-+;--) - \mu(--;++) - \mu(--;+-) + \mu(--;-+) + \mu(--;--), \quad (5)$$

and similarly for $E(a,b')_{lhv}$, $E(a',b)_{lhv}$, and $E(a',b')_{lhv}$.

Now let us take a linear combination of these four correlation functions, utilizing an arbitrary assignment of three coefficients 1 and one coefficient - 1, or alternatively of three coefficients -1 and one coefficient 1. An example is $E(a,b)_{lhv} - E(a,b')_{lhv} + E(a',b)_{lhv} + E(a',b')_{lhv}$. A term $\mu(.\ .\ ;\ .\ .)$ occurs four times in this linear combination. Let us count the number of agreements in sign between the pair of arguments to the left of the semicolon and the pair to the right of the semicolon. Obviously, there can be either four or two or zero agreements. For example, the terms $\mu(++;++)$, $\mu(++;+-)$, and $\mu(++;--)$ exhibit four, two, and zero agreements respectively. If four, the net coefficient of the term in the linear combination taken as an example is 2. If zero, the net coefficient is -2. If two, the coefficient is +2 in somes cases and -2 in others. Consequently,

$$E(a,b)_{lhv} - E(a,b')_{lhv} + E(a',b)_{lhv} + E(a',b')_{lhv}$$

$$\leq 2\left[\mu(++;++) + \mu(++;+-) + \ldots\ldots + \mu(--;--)\right] = 2. \quad (6)$$

(The sum of the $\mu(..;..)$ is 1 because the subspaces are mutually disjoint and exclusive, and μ is a probability distribution.)

More generally,

$$\underbrace{E(a,b)_{lhv} \quad E(a,b')_{lhv} \quad E(a',b)_{lhv} \quad E(a',b')_{lhv}} \leq 2. \quad (6')$$

3 plus signs, 1 minus sign

or

3 minus signs, 1 plus sign.

This completes the proof. We have an inequality, or a family of inequalities, which holds for every theory which is local in Bell's sense and for every distribution μ. These inequalities appear to impose rather weak constraints upon the correlation function E(a,b). Naturally, in view of the generality of the reasoning, they do not specify the value of E(a,b) for a single pair of arguments a,b. Furthermore, it appears to be difficult to express the intuitive content of the inequalities. Their interest lies in the fact that they hold for the entire family of local hidden variables theories and yet, as will be seen below, they are violated in certain circumstances by quantum mechanics. Although the proof is simple, it may be somewhat obscure where the assumption of locality has been used. The answer is: in the partition of Ω into the sexteen subspaces. In saying, for example, that a measurement of A_a would yield the result 1 if λ lies in the subspace (+-;-+), we did not specify that B_b or $B_{b'}$, or neither of these, would be measured at the same time as A_a. In other words, we assumed that the value of A_a depended only upon λ and not upon the choice by the experimenter of a member of the family of observables $\{B_b\}$.

In order to see the statistical disagreement of quantum mechanics with each local hidden-variable theory -- i.e., the falsity of Eq. (2) -- let us consider a photon pair in the quantum state Ψ defined by Eq. (1), and let A_a and B_b have the meanings assigned immediately after Eq. (3). The quantum mechanical operator corresponding to A_a is

$$\begin{pmatrix} \cos 2a & \sin 2a \\ \sin 2a & -\cos 2a \end{pmatrix},$$

and it is easily verified that the vectors $\begin{pmatrix} \cos a \\ \sin a \end{pmatrix}$ and $\begin{pmatrix} -\sin a \\ \cos a \end{pmatrix}$ are eigenvectors of this operator with eigenvalues 1 and -1 respectively; similarly for B_b. One can then calculate that

$$E(a,b)_{qm} \equiv E(\alpha)_{qm} \equiv \langle \Psi | A_a \cdot B_b | \Psi \rangle = \cos 2\alpha , \quad (7)$$

where α is defined as the least angle between the undirected line determined by a and the undirected line determined by b. (The reason for considering the undirected rather than the directed line is that we are dealing with the linear polarization of photons rather than quantities such as electron spins.) Now choose $a = 0$, $b = \pi/8$, $a' = \pi/4$, and $b' = 3\pi/8$ as in figure 1.

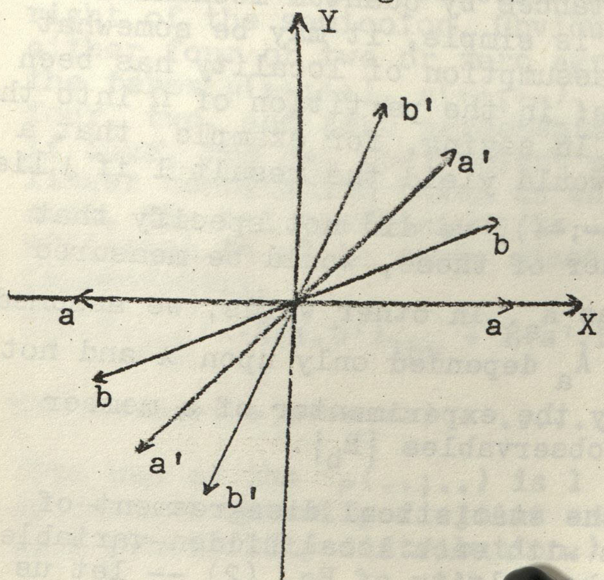

fig. 1

Then

$$E(a,b)_{qm} = E(a',b)_{qm} = E(a',b')_{qm} = -E(a,b')_{qm} = \sqrt{2}/2,$$

and therefore

$$E(a,b)_{qm} - E(a,b')_{qm} + E(a',b)_{qm} + E(a',b')_{qm} = 2\sqrt{2}, \quad (8)$$

in evident disagreement with inequality (6).

An additional supposition makes possible the demonstration of another striking disagreement between quantum mechanics and local hidden-variable theories. Suppose that $E(a,b)_{lhv}$ depends only upon α and can therefore be written $E(\alpha)_{lhv}$. (A theory of local hidden variables for which this supposition is false would already conflict with the quantum mechanical prediction (7). More importantly, the supposition is experimentally testable.) For the above choice of angles, $E(a,b)_{lhv} = E(a',b)_{lhv} = E(a',b')_{lhv}$ $= E(\pi/8)_{lhv}$, while $E(a,b')_{lhv} = E(3\pi/8)_{lhv}$, so that Ineq. (6) becomes

$$3E(\pi/8)_{lhv} - E(3\pi/8)_{lhv} \leq 2. \quad (9a)$$

Now make another choice of angles, and let a, b, a', b' be respectively 0, $3\pi/8$, $3\pi/4$, and $9\pi/8$ as in figure 2

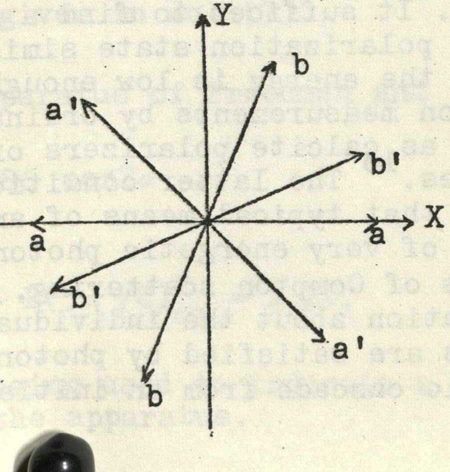

fig. 2

Substitution into the appropriate specification of Ineq. (6') yields

$$3E(\pi/8)_{lhv} - E(3\pi/8)_{lhv} \leq 2. \qquad (9b)$$

From (9a) and (9b) one obtains the surprising inequality

$$E(\pi/8)_{lhv} - E(3\pi/8)_{lhv} \leq 1, \qquad (10)$$

which was first derived by S.Freedman.[9] The disagreement with quantum mechanics is clear, since $E(\pi/8)_{qm} = -E(\pi/8)_{qm} = \sqrt{2}/2$, so that

$$E(\pi/8)_{qm} - E(3\pi/8)_{qm} = \sqrt{2} \quad . \qquad (11)$$

This disagreement between Eq. (8) and Ineq. (6) shows that quantum mechanics is irreconcilable with a view of the world in which (i) locality holds and (ii) each spatially distinct physical system has an intrinsic state independent of observation. Since Bohr (see Reference 3) rejected proposition (ii), one could properly say that the foregoing analysis gives support to Bohr's interpretation of quantum mechanics.

The road is now open towards an experimental test of the entire family of local hidden-variable theories. It suffices to find a source of photon pairs in a polarization state similar to Ψ, and such that the energy is low enough to permit polarization measurements by ordinary optical means, such as calcite polarizers or piles of glass plates.[10] The latter condition is required by the fact that typical means of analyzing the polarization of very energetic photons, for instance, by means of Compton scattering, yield very little information about the individual photon. The conditions are satisfied by photon pairs emitted in an atomic cascade from an initial J=0 state to a final

J=0 state via an intermediate J=1 state. The total angular momenta of the two photons are precisely correlated, because $(j_1+j_2)\cdot\hat{n} = 0$ for each $\hat{n}$. In order to obtain a precise correlation of the polarizations, one need only insert two diaphragms with infinitesimal apertures, restricting the direction of the first photon to the immediate neighborhood of z and that of the second photon to the immediate neighborhood of -z. Then the contribution of the orbital angular momentum would vanish, leaving $(\underline{s}_1+\underline{s}_2)\cdot\hat{n} = 0$. Mathematically, this result is equivalent to the state Ψ. Unfortunately, infinitesimal apertures permit only an infinitesimal flux of photons. A practical experiment requires finite apertures, say of half-angle θ. If θ is sufficiently small, the contribution of the orbital angular momentum does not greatly diminish the correlation of the polarizations; the expression for $E(a,b)_{qm}$ is Eq. (7) must be multiplied by $F(\theta)$, where F is a monotonically decreasing function which is 1 for $\theta = 0$ and .99 for $\theta = 30^o$. A further complication is the non-existence of ideal polarization analyzers. Each actual analyzer absorbs a small percentage of photons polarized along its axis of polarization and permits the passage of a small percentage of photons polarized perpendicular to that axis. The imperfection of the analyzers causes a further deviation of $E(a,b)_{qm}$ from the ideal value given in Eq. (7).

In the case of the apparatus of Freedman and Clauser[11],

$$E(\alpha)^r_{qm} = .86 \cos 2\alpha \quad , \tag{12}$$

and therefore

$$E(\pi/8)^r_{qm} - E(3\pi/8)^r_{qm} = 1.22, \tag{13}$$

the superscript "r" being used to indicate a realistic description of the apparatus.

Since Eq. (13) disagrees with inequality (10), the experimental results must refute either quantum mechanics or the family of local hidden-variable theories (or possibly both). Freedman and Clauser found the following experimental result:

$$E(\pi/8)_{exp} - E(3\pi/8)_{exp} = 1.20 \pm .032, \qquad (14)$$

in excellent agreement with the predictions of quantum mechanics and in sharp disagreement with those of local hidden-variable theories. Their conclusion was strengthened by measurements of $E(\alpha)_{exp}$ for a large number of angles α, with results which confirmed Eq. (12).

Is the story of local hidden-variable theories finished? In our opinion, not entirely. First of all, too much is at stake to remain content with the single experiment of Freedman and Clauser. There has, in fact, been another experiment, that of Kasday, Ullman, and Wu[12], which also confirmed quantum mechanics and disagreed with local hidden-variable theories; but this experiment is less decisive than that of Freedman and Clauser, since it relies upon an additional assumption. It is to be hoped that further experiments of the foregoing type will be performed in the near future, in order to check the results obtained so far.
Secondly, a dedicated advocate of hidden-variable theories could be skeptical about the validity of Bell's condition of locality. The non-occurence of action-at-a-distance implies Bell's condition only in a very special case: when the choice of the apparatus adapted to measuring a member of the family $\{B_b\}$ and the event of measuring a member of the family $\{A_a\}$ are events with a space-like separation. But that is not the case in the experiment of Freedman and Clauser, who leave their polariza-

tion analyzers in a fixed orientation for a period of several minutes. As a result, there is in principle sufficient time for information concerning the orientation of the analyzers to be transmitted from one to the other without violating the relativistic restriction upon the velocity of signals. If, however, the orientation of the analyzers were not fixed before the propagation of the photons, then there would not be time for such transmission of information, and Bell's condition of locality would have to be accepted. Clauser has ingeniously suggested that control of the orientation, while the photons are in flight, could be effected by means of a Kerr cell, which establishes an axis of polarization by means of an imposed electric field. The direction fo the field, in turn, could be determined by some chance process.

One could also imagine that the coincidences of events registered by the photo-detectors are not exact indications of the correlated polarizations of the photon pairs. Since the detectors employed by Freedman and Clauser had respective efficiencies of 13% and 28%, fever than 4% of the photon pairs emerging from the analyzers were counted. Clauser and Horne have constructed a model, which is very artificial and nevertheless strictly local, in which the predicitions concerning the coincidence-count agrees with those of quantum mechanics. In their model, the particles emerging from the analyzers obey the inequalities of local hidden-variable theories, but registration by the detectors is selective in a very deceptive manner. One could hardly believe that such a model is realized in nature, for it supposes a kind of conspiracy. Nevertheless, the mathematical consistency of the model shows that we need an experiment yet more definitive than that of Freedman and Clauser. Probably the best procedure would be the following: to seek a kind of molecule which could be excited to an unstable state of total angular momentum 0 and which would then divide spontaneously into two neutral parts, each of total angular momentum $\frac{1}{2}$. Each of the two parts could be examined by means of a Stern-Gerlach apparatus, and most of the pairs emerging from the two analyzers could be detected. As a result, the advocate of local hidden-variable theories could not conjecture that the success of the quantum mechanical predictions would be the result of the conspiratorial behavior of the detectors .

We should say, in conclusion, that if these proposed new experiments were performed, we are confident that the quantum mechanical predictions again would be confirmed. Our confidence does not, however, imply satisfaction with current philosophical interpretations of quantum mechanics. We believe that the problem of measurement and the paradox of Einstein, Podolsky, and Rosen have not yet been satisfactorily resolved. However, we now know that the solution to the anomalies of quantum mechanics requires a more subtle and profound revision of our view of nature than that which was offered by local hidden-variable theories. In particular, if the truth is to be found among non-local hidden-variable theories, then our view of space-time structure will have to be radically modified.

<u>Postscript,</u> September 27, 1973 :

Experimental results in conflict with those of Freedman and Clauser have recently been found by R.A.Holt and are reported in his unpublished doctoral dissertation, <u>Atomic Cascade Experiments</u>, submitted to the Department of Physics of Harvard University in 1973. He carried out the design of Reference 10, as they did, but with a different atomic source of photon pairs (^{198}Hg) and different optical arrangements. Holt's data agreed with Ineq. (6), and his measurement of

$E(\pi/8)_{exp} - E(3\pi/8)_{exp}$ was four standard deviations away from the predicted quantum mechanical value. Although neither Holt nor other experimentalists have been able to find systematic errors in his work, he remains cautious about the correctness of his findings.-

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5) Professor Rosen has stated in conversations and correspondence that Einstein anticipated an almost exact agreement of the statistical predictions of quantum mechanics and of a more complete theory within the ordinary domain of atomic physics, though he expected deviations in domains remote from those in which quantum mechanics was initially successful. The correlation phenomena considered in this article surely fall within the domain of atomic physics.

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