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Beables for Quantum Field Theory

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Dedicated to Professor D. Bohm

1. Introduction

Bohm's 1952 papers[1,2] on quantum mechanics were for me a revelation. The elimination of indeterminism was very striking. But more important, it seemed to me, was the elimination of any need for a vague division of the world into "system" on the one hand, and "apparatus" or "observer" on the other. I have always felt since that people who have not grasped the ideas of those papers ... and unfortunately they remain the majority ... are handicapped in any discussion of the meaning of quantum mechanics.

When the cogency of Bohm's reasoning is admitted, a final protest is often this: it is all nonrelativistic. This is to ignore that Bohm himself, in an appendix to one of the 1952 papers,[2] already applied his scheme to the electromagnetic field. And application to scalar fields is straightforward.[3] However until recently,[4,5] to my knowledge, no extension covering Fermi fields had been made. Such an extension will be sketched here. The need for Fermi fields might be questioned. Fermions might be composite structures of some kind.[6] But also they might not be, or not all. The present exercise will not only include Fermi fields, but even give them a central role. The dependence on the ideas of de Broglie[7] and Bohm,[1,2] and also on my own simplified extension to cover spin,[8,9,10] will be manifest to those familiar with these things. However no such familiarity will be assumed.

A preliminary account of these notions[5] was entitled "Quantum field theory without observers, or observables, or measurements, or systems, or apparatus, or wavefunction collapse, or anything like that". This could suggest to some that the issue in question is a philosophical one. But I insist that my concern is strictly professional. I think that conventional formulations of quantum theory, and of quantum field theory in particular, are unprofessionally vague and ambiguous. Professional theoretical physicists ought to be able to do better. Bohm has shown us a way.

It will be seen that all the essential results of ordinary quantum field theory are recovered. But it will be seen also that the very sharpness of the reformulation brings into focus some awkward questions. The construction of the scheme is not at all unique. And Lorentz invariance plays a strange, perhaps incredible role.

2. Local Beables

The usual approach, centred on the notion of "observable", divides the world somehow into parts: "system" and "apparatus". The "apparatus" interacts from time to time with the "system", "measuring" "observables". During "measurement" the linear Schrödinger evolution is suspended, and an ill-defined "wavefunction collapse" takes over. There is nothing in the mathematics to tell what is "system" and what is "apparatus", nothing to tell which natural processes have the special status of "measurements". Discretion and good taste, born of experience, allow us to use quantum theory with marvelous success, despite the ambiguity of the concepts named above in quotation marks. But it seems clear that in a serious fundamental formulation such concepts must be excluded.

In particular we will exclude the notion of "observable" in favour of that of "*be*able". The beables of the theory are those elements which might correspond to elements of reality, to things which exist. Their existence does not depend on "observation". Indeed observation and observers must be made out of beables.

I use the term "beable" rather than some more committed term like "being"[11] or "beer"[12] to recall the essentially tentative nature of any physical theory. Such a theory is at best a *candidate* for the description of nature. Terms like "being", "beer", "existent" [11,13] etc., would seem to me lacking in humility. In fact "beable" is short for "maybe-able".

Let us try to promote some of the usual "observables" to the status of beables. Consider the conventional axiom:

the probability of observables $(A, B, \ldots)$ (1)
if observed at time t
being observed to be $(a, b, \ldots)$
is

$$\sum_q |\langle a, b, \ldots q|t\rangle|^2$$

where q denotes additional quantum numbers which
together with the eigenvalues $(a, b, \ldots)$
form a complete set.

This we replace by

the probability of beables $(A, B, \ldots)$ (2)
at time t
being $(a, b, \ldots)$
is

$$\sum_q |\langle a, b, \ldots q|t\rangle|^2$$

where q denotes additional quantum numbers which
together with the eigenvalues $(a, b, \dots)$
form a complete set.

Not all "observables" can be given beable status, for they do not all have simultaneous eigenvalues, i.e. do not all commute. It is important to realize therefore that most of these "observables" are entirely redundant. What is essential is to be able to define the positions of things, including the positions of instrument pointers or (the modern equivalent) of ink on computer output.

In making precise the notion "positions of things" the energy density $T_{00}(x)$ comes immediately to mind. However the commutator

$$[T_{00}(x), T_{00}(y)]$$

is not zero, but proportional to derivatives of delta functions. So the $T_{00}(x)$ do not have simultaneous eigenvalues for all x. We would have to devise some new way of specifying a joint probability distribution.

We fall back then on a second choice – fermion number density. The distribution of fermion number in the world certainly includes the positions of instruments, instrument pointers, ink on paper, ... and much much more.

For simplicity we replace the three-space continuum by a dense lattice, keeping time t continuous (and real!). Let the lattice points be enumerated by

$$l = 1, 2, \dots L$$

where L is very large. Define lattice point fermion number operators

$$\psi^+(l)\psi(l)$$

where summation over Dirac indices and over all Dirac fields is understood. The corresponding eigenvalues are integers

$$F(l) = 1, 2, \dots 4N$$

where N is the number of Dirac fields. The fermion number configuration of the world is a list of such integers

$$n = (F(1), F(2), \dots F(L))$$

We suppose the world to have a definite such configuration at every time t:

$$n(t)$$

The lattice fermion number are the local beables of the theory, being associated with definite positions in space. The state vector $|t\rangle$ also we consider as a beable, although not a local one. The complete specification of our world at time t is then a combination

$$(|t\rangle, n(t)) \tag{3}$$

It remains to specify the time evolution of such a combination.

3. Dynamics

For the time evolution of the state vector we retain the ordinary Schrödinger equation,

$$d/dt|t\rangle = -iH|t\rangle \tag{4}$$

where H is the ordinary Hamiltonian operator.

For the fermion number configuration we prescribe a stochastic development. In a small time interval dt configuration m jumps to configuration n with transition probability

$$dt\, T_{nm} \tag{5}$$

where

$$T_{nm} = J_{nm}/D_m \tag{6}$$

$$J_{nm} = \sum_{qp} 2\,\mathrm{Re}\langle t|nq\rangle\langle nq| - iH|mp\rangle\langle mp|t\rangle \tag{7}$$

$$D_m = \sum_q |\langle mq|t\rangle|^2 \tag{8}$$

provided $J_{nm} > 0$, but

$$T_{nm} = 0 \quad \text{if} \quad J_{nm} \leqslant 0 \tag{9}$$

From (5) the evolution of a probability distribution P_n over configurations n is given by

$$d/dt\, P_n = \sum_m (T_{nm}P_m - T_{mn}P_n) \tag{10}$$

Compare this with a mathematical consequence of the Schrödinger equation (4):

$$d/dt|\langle nq|t\rangle|^2 = \sum_{mp} 2\,\mathrm{Re}\langle t|nq\rangle\langle nq| - iH|mp\rangle\langle mp|t\rangle$$

or

$$d/dt\, D_n = \sum_m J_{nm} = \sum_m (T_{nm}D_m - T_{mn}D_n) \tag{11}$$

If we assume that at some initial time

$$P_n(0) = D_n(0) \tag{12}$$

then from (11) the solution of (10) is

$$P_n(t) = D_n(t) \tag{13}$$

Envisage then the following situation. In the beginning God chose 3-space and 1-time, a Hamiltonian H, and a state vector $|0\rangle$. Then She chose a fermion configuration $n(0)$. This She chose at random from an ensemble of possibilities with distribution $D(0)$ related to the already chosen state vector $|0\rangle$. Then She left the world alone to evolve according to (4) and (5).

It is notable that although the probability distribution P in (13) is governed by D and so by $|t\rangle$, the latter is not to be thought of as just a way of expressing the probability distribution. For us $|t\rangle$ is an independent beable of the theory. Otherwise its appearance in the transition probabilities (5) would be quite unintelligible.

The stochastic transition probabilities (5) replace here the deterministic guiding equation of the de Broglie–Bohm "pilot wave" theory. The introduction of a stochastic element, for beables with discrete spectra, is unwelcome, for the reversibility[14] of the Schrödinger equation strongly suggests that quantum mechanics is not fundamentally stochastic in nature. However I suspect that the stochastic element introduced here goes away in some sense in the continuum limit.

4. OQFT and BQFT

OQFT is "ordinary" "orthodox" "observable" quantum field theory, whatever that may mean. BQFT is de Broglie–Bohm beable quantum field theory. To what extent do they agree? The main difficulty with this question is the absence of any sharp formulation of OQFT. We will consider two different ways of reducing the ambiguity.

In OQFT1 the world is considered as one big experiment. God prepared it at the initial time $t = 0$, and let it run. At some much later time T She will return to judge the outcome. In particular She will observe the contents of all the physics journals. This will include of course the records of our own little experiments — as distributions of ink on paper, and so of fermion number. From (13) the OQFT1 probability D that God will observe one configuration rather than another is identical with the BQFT probability P that the configuration *is* then one thing rather than another. In this sense there is complete agreement between OQFT1 and BQFT on the result of God's big experiment — including the results of our little ones.

OQFT1, in contrast with BQFT, says nothing about events in the system in between preparation and observation. However adequate this may be from an Olympian point of view, it is rather unsatisfactory for us. We live in between creation and last judgement — and imagine that we experience events. In this respect another version of OQFT is more appealing. In OQFT2, whenever the state can be resolved into a sum of two (or more) terms

$$|t\rangle = |t, 1\rangle + |t, 2\rangle \tag{14}$$

which are "macroscopically different", then in disregard for the Schrödinger equation the state "collapses" somehow into one term or the other:

$$\begin{aligned} |t\rangle &\to N_1^{-1/2}|t, 1\rangle \text{ with probability } N_1 \\ |t\rangle &\to N_2^{-1/2}|t, 2\rangle \text{ with probability } N_2 \end{aligned} \tag{15}$$

where

$$N_1 = |\langle t, 1|t, 1\rangle| \quad N_2 = |\langle t, 2|t, 2\rangle| \tag{16}$$

In this way the state is always, or nearly always, macroscopically unambiguous and defines a macroscopically definite history for the world. The words "macroscopic" and "collapse" and terribly vague. Nevertheless this version of OQFT is probably the nearest approach to a rational formulation of how we use quantum theory in practice.

Will OQFT2 agree with OQFT1 and BQFT at the final time T? This is the main issue in what is usually called "the Quantum Measurement Problem". Many authors, analyzing many models, have convinced themselves that the state vector collapse of OQFT2 is consistent with the Schrödinger equation of OQFT1 "for all practical purposes".[15] The idea is that even when we retain both components in (13), evolving as required by the Schrödinger equation, they remain so different as not to interfere in the calculation of anything of interest. The following sharper form of this hypothesis seems plausible to me: the macroscopically distinct components remain so different, for a very long time, as not to interfere in the calculation of D and $J^{(5)}$. In so far as this is true, the trajectories of OQFT2 and BQFT will agree macroscopically.

5. Concluding Remarks

We have seen that BQFT is in complete accord with OQFT1 as regards the final outcome. It is plausibly consistent with OQFT2 in so far as the latter is unambiguous. BQFT has the advantage over OQFT1 of being relevant at all times, and not just at the final time. It is superior to OQFT2 in being completely formulated in terms of unambiguous equations.

Yet even BQFT does not inspire complete happiness. For one thing there is nothing unique about the choice of fermion number density as basic local beable. We could have others instead, or in addition. For example the Higg's fields of contemporary gauge theories could serve very well to define "the positions of things". Other possibilities have been considered by K. Baumann.[4] I do not see how this choice can be made experimentally significant, so long as the final results of experiments are defined so grossly as by the positions of instrument pointers, or of ink on paper.

And the status of Lorentz invariance is very curious. BQFT agrees with OQFT on the result of the Michelson–Morley experiment, and so on. But the formulation of BQFT relies heavily on a particular division of space-time into space and time. Could this be avoided?

There is indeed a trivial way of imposing Lorentz invariance.[4] We can imagine the world to differ from vacuum over only a limited region of infinite Euclidean space (we forget general relativity here). Then an overall centre of mass system is defined. We can simply assert that our equations hold in this centre of mass system. Our scheme is then Lorentz invariant. Many others could be made Lorentz invariant in the same way ... for example Newtonian mechanics. But such Lorentz invariance would not imply a null result for the Michelsen–Morley experiment ...

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which could detect motion relative to the cosmic mass centre. To be predictive, Lorentz invariance must be supplemented by some kind of locality, or separability, consideration. Only then, in the case of a more or less isolated object, can motion relative to the world as a whole be deemed more or less irrelevant.

I do not know of a good general formulation of such a locality requirement. In classical field theory, part of the requirement could be formulation in terms of differential (as distinct from integral) equations in $3+1$ dimensional space-time. But it seems clear that quantum mechanics requires a much bigger configuration space. One can formulate a locality requirement by permitting arbitrary external fields, and requiring that variation thereof have consequences only in their future light cones. In that case the fields could be used to set measuring instruments, and one comes into difficulty with quantum predictions for correlations related to those of Einstein, Podolsky, and Rosen.[18] But the introduction of external fields is questionable. So I am unable to prove, or even formulate clearly, the proposition that a sharp formulation of quantum field theory, such as that set out here, must disrespect serious Lorentz invariance. But it seems to me that this is probably so.

As with relativity before Einstein, there is then a preferred frame in the formulation of the theory ... but it is experimentally indistinguishable.[20,21,22] It seems an eccentric way to make a world.

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