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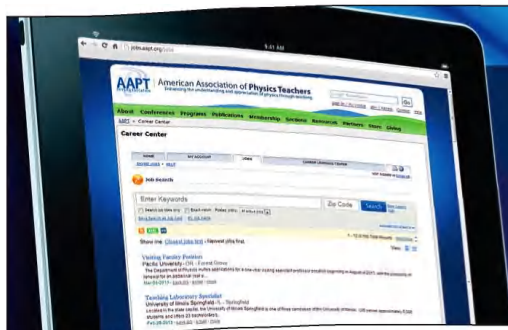
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# Einstein's Interpretation of Quantum Mechanics

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*Einstein's arguments concerning the interpretation of quantum mechanics are reviewed and contrasted with certain misconceptions regarding his attitude toward the theory. He considered Born's statistical interpretation to be the only satisfactory one, and he was not a supporter of hidden-variable theories such as that of Bohm. His criticism of the interpretation accepted, at least tacitly, by many physicists was that the quantum state function does not provide a description of an individual system but rather of an ensemble of similar systems. This criticism was not based merely upon his famous remark that God does not play dice, but upon some definite physical arguments which did not assume determinism.*

## INTRODUCTION

What was Einstein's attitude toward the statistical quantum theory? A widespread myth (difficult to trace but quite real) holds that he misunderstood or rejected quantum mechanics on rather vague philosophical grounds, that he disregarded the statistical quantum theory merely because he did not believe in a God who plays dice. In a preface to a British Broadcasting Corporation series<sup>1</sup> about Einstein, Christopher Sykes speaks of, "The main and most pernicious [myth] concerns the last phase of his life . . ." (the period spent in the United States of America) when he was sometimes regarded as a man of enfeebled energy and even decaying mind.

That Einstein criticized the Copenhagen interpretation of quantum mechanics is well known, although the nature of his critical arguments and

his own interpretation of the theory are less well known. Indeed, Heisenberg's essay "The Development of the Interpretation of the Quantum Theory,"<sup>2</sup> in which he replied in detail to many critics of the Copenhagen interpretation, takes no account of Einstein's specific arguments. Einstein is said merely to belong to a group which "expresses rather its general dissatisfaction with the quantum theory, without making definite counterproposals, either physical or philosophical in nature." We shall see that this latter remark is certainly false.

Einstein's attitude is best expressed in his "Reply to Criticisms,"<sup>3</sup> to which we shall return later. There he sums up his argument as follows (p. 671):

One arrives at very implausible theoretical conceptions, if one attempts to maintain the thesis that the statistical quantum theory is in principle capable of producing a complete description of an individual physical system. On the other hand, those difficulties of theoretical interpretation disappear, if one views the quantum-mechanical description as the description of ensembles of systems.

We see here that he did indeed make a definite counterproposal to the Copenhagen interpretation, and that he regarded quantum mechanics as a satisfactory theory *provided* one gives up the claim that a quantum state function constitutes a complete description of an *individual* system.

How, then, could such ignorance, misinformation, and pernicious myths about Einstein have come to be? Presumably it is because he did not participate actively in the development of the new quantum mechanics (after 1925) except as a critic, and because he did not publish a systematic account of quantum theory from his own point of view. But from his publications and his correspondence with contemporary physicists, we can obtain a clear picture of his attitude toward quantum mechanics.

We shall frequently refer to *The Born-Einstein Letters* (BEL)<sup>4</sup> and to *Letters on Wave Mechanics* (LWM).<sup>5</sup> A useful bibliography of Einstein's

writing is contained in Ref. 3, and a more complete list has been compiled by Boni, Russ, and Lawrence.<sup>6</sup>

### FIRST IMPRESSION

Modern quantum mechanics originated in 1925 from two independent starting points, Heisenberg's "matrix mechanics" and Schrödinger's "wave mechanics." Although the two formalisms were shown to be equivalent by Schrödinger in 1926, they appeared to be very different in their original form.

Now Einstein had earlier suggested that Louis deBroglie's thesis, which postulated waves associated with material particles, should be taken seriously. So it is not at all surprising that he responded favorably to Schrödinger's paper. Somewhat surprising is his remark,<sup>7</sup> "I am convinced that you have made a decisive advance with your formulation of the quantum condition, just as I am equally convinced that the Heisenberg-Born route is off the track."<sup>7</sup>

Here he was referring to the fact that, if two noninteracting systems are regarded as one system, then their energies are clearly additive according to Schrödinger's theory. Einstein believed (incorrectly) that the same was *not* true in the matrix formalism. We may presume that he was not aware of Schrödinger's demonstration<sup>8</sup> of equivalence between the two formalisms because that paper was published on 4 May 1926.

Earlier, in a letter<sup>9</sup> to Mrs. Born dated 7 March 1926, Einstein had said, "The Heisenberg-Born concepts leave us all breathless, and have made a deep impression on all theoretically oriented people." But on 4 December 1926 he wrote to Born,<sup>10</sup> "Quantum mechanics is certainly imposing. But an inner voice tells me that it is not yet the real thing. The theory says a lot, but does not really bring us any closer to the secret of the 'old one.' I, at any rate, am convinced that He is not playing at dice." No doubt the publication of Schrödinger's work between these two letters influenced Einstein to change his attitude, but since the equivalence of the two formalisms had been demonstrated this is hardly a sufficient reason. Here we must remember that a formalism requires a *physical interpretation* before it becomes a theory. Schrödinger interpreted the  $|\psi|^2$  as a real extended charge distribution which evolves

continuously, whereas Born gave an interpretation in terms of discrete events and transition probabilities. In fact, the interpretation of quantum theory was not at all settled at that time and it was to be the main topic of discussion at the Solvay Conference the following year.

### THE SOLVAY CONFERENCE

The Fifth Solvay Conference, held in Brussels from 24 to 29 October 1927, provided an opportunity for the leading physicists of the day to discuss the new quantum theory. Einstein's only contribution to the official proceedings was one comment in the general discussion.<sup>11</sup> He considered the situation pictured in Fig. 1. Let  $S$  be a screen with a small opening  $O$ , and let  $P$  be a photographic film in the form of a large hemisphere. Suppose that electrons fall on  $S$  and that some of them pass through  $O$ . Because of the smallness of the opening  $O$ , the deBroglie-Schrödinger wave appropriate to the quantum mechanical description will be diffracted at  $O$ , and a spherical wave will propagate towards  $P$ .

Einstein then distinguished two possible points of view.

Idea I:<sup>12</sup> "The deBroglie-Schrödinger waves do not correspond to a single electron, but to a cloud of electrons extended in space. The theory does not give any information about the individual processes, but only about the ensemble of an infinity of elementary processes."

Idea II: "The theory has the pretention to be a complete theory of individual processes." Although the particle may initially be described by a small wave packet, the wave will be diffracted at  $O$ , and will cover the whole of the film  $P$ .

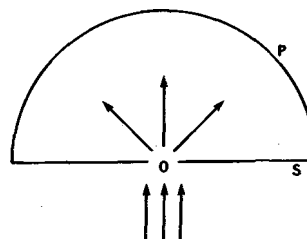


FIG. 1. Electrons incident on screen  $S$  at  $O$  are diffracted and recorded on the photographic film  $P$ .

According to Idea I, purely statistical,  $|\psi|^2$  expresses the probability that a particle of the cloud should exist at a particular place.

According to Idea II,  $|\psi|^2$  expresses the probability that at a certain instant *one and the same* particle is found at the specified place. Hence it should be possible for *one* particle to be detected at *two or more* places on the film. This objectionable conclusion can be avoided only if we suppose there is a very peculiar action-at-a-distance mechanism which prevents the extended wave from producing an action at more than one place on the film. But this would contradict special relativity.

Einstein concluded, "In my opinion, one can only remove this objection in this manner, that one does not describe only the process by the Schrödinger wave, but at the same time one localizes the particle during the propagation." In other words, he argued that a particle should always be thought of as possessing a definite though perhaps unknown position, even when no such definite position is described by the wave function  $\psi$ .

Einstein's argument had certain weaknesses. He conceded unnecessarily (and actually incorrectly) that only Idea II can account for the validity of conservation laws in elementary processes, the result of the Geiger-Bothe experiment, and the occurrence of tracks in the Wilson cloud chamber. More seriously, in Idea I he speaks of "a cloud of electrons", suggesting that a wave function in a space of three dimensions refers to a many electron system rather than to an ensemble (in the Gibbian sense) of one electron systems. It is this latter interpretation, which I have discussed in detail elsewhere,<sup>13</sup> that he was to adopt consistently in later years. Nevertheless, he should be credited with identifying in Idea II the necessity for a paradoxical *reduction of the wave function*, which was to occupy the attention of theorists concerned with the quantum theory of measurement for several decades to come.

Outside of the official discussions there was an informal debate between Bohr and Einstein, which is probably more famous than the formal proceedings of the meeting. As Bohr<sup>14</sup> later described, it concerned the possibility or impossibility of simultaneously determining two canonical variables (such as position and momentum, or energy and time) with a greater precision than

that allowed by Heisenberg's uncertainty relations. Although Einstein did not succeed in producing an example, it is known today that in certain situations it is possible to measure two noncommuting observables simultaneously with a precision unrestricted by Heisenberg's relations, but contrary to the belief held in 1927, the foundations of quantum mechanics are *not* shaken by this fact.<sup>15</sup> For quantum mechanics, properly understood, does not prohibit or restrict simultaneous measurement of noncommuting observables, but rather it does not deal with such measurements at all.<sup>16</sup> Therefore, we shall pass by the great Bohr-Einstein debate. Although it played a significant historical role in gathering acceptance for the Copenhagen interpretation, it dealt with an issue which is not central to the structure of quantum theory.

#### CAN THE QUANTUM-MECHANICAL DESCRIPTION OF PHYSICAL REALITY BE CONSIDERED COMPLETE?

The next significant development was the publication in 1935 of a paper by Einstein, Podolsky, and Rosen (EPR) with the above title.<sup>17</sup> At the outset they stated that the *correctness* of quantum mechanical predictions would be taken for granted, and only the *completeness* of the description of reality (i.e., by a wave function) would be questioned.

Two definitions were introduced.

D1. A *necessary* condition for a *complete* description is that "every element of the physical reality must have a counterpart in the physical theory."

D2. A *sufficient* criterion for identifying an *element of reality* is, "If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity."

They then considered two particles which were correlated by virtue of having both interacted with the same piece of apparatus in the past, but which were no longer interacting either directly or indirectly. It is possible, assuming the correctness of quantum mechanics, that the two-particle wave function should be an eigenfunction of

$q_2 - q_1 [=x_0, \text{ for example}]$  and  $p_1 + p_2 [=0, \text{ say}]$ , since the difference of the position operators for particles 1 and 2 commutes with the sum of the momentum operators.

If we measure the position of particle 1 to be  $q_1 = x$ , then we immediately deduce that the position of particle 2 must be  $x + x_0$ . Since this measurement did not disturb particle 2 in any way, we conclude from D2 that  $q_2 = x + x_0$  is an element of reality, i.e., that particle 2 has a definite value of position which was merely unknown before the measurement. On the other hand, if we measure the momentum of particle 1 then we deduce, by an analogous argument, that  $p_2$  is an element of reality.

But if two noncommuting observables like  $q_2$  and  $p_2$  can have simultaneous reality, then clearly the description of the system by a wave function is incomplete, for no wave function can be an eigenfunction of both  $q_2$  and  $p_2$ .

The authors anticipated the possible counterproposal that two physical quantities be regarded as simultaneous elements of reality *only when they can be simultaneously measured or predicted*. But this would make the reality of  $p_2$  and  $q_2$  depend upon a measurement carried out on particle 1, which does not disturb particle 2 in any way. "No reasonable definition of reality could be expected to permit this," they asserted.

I shall not elaborate the responses to and refinements of the EPR argument which have been published, since I have discussed the most important of these elsewhere.<sup>18</sup> For example, Bohr said, in essence, that although the measurement on particle 1 does not disturb particle 2 in a mechanical way, it does influence the possible types of *predictions* which we can make about the future behavior of particle 2. This is certainly true but it is in no sense a refutation of EPR. In addition to published comments, Einstein received several letters pointing out just why the argument was "wrong", but to his amusement the writers did not agree among themselves about the nature of the "error"! The reason for this sort of response is most likely that a belief in the completeness and finality of quantum mechanics was the then accepted fashion. However, it should be noted that, although EPR explicitly accepted the correctness of quantum mechanical predictions, the interpretation of the theory which they

regarded as acceptable is contained only implicitly in their paper. Thus, even though quantum mechanics itself (to be distinguished from certain tenants of the Copenhagen interpretation) was not under attack, some physicists may have been unsure just what would be the status of quantum theory if the conclusion of EPR were accepted.

### THE STATISTICAL ENSEMBLE INTERPRETATION

In a long article entitled "Physics and Reality,"<sup>19</sup> in which he discussed the relationships of the major chapters of theoretical physics to each other and to the world which they describe, Einstein first gave an interpretation of quantum mechanics which may be taken as definitive of his opinion.

After a brief discussion of the accomplishments of quantum theory, he then presented a new argument in support of his view that it does not, in general, describe the individual system. Temporarily leaving open the question of the extent to which a particular stationary state  $\psi_r$  of the Schrödinger equation might provide a complete description of a physical system, he considered the following problem. A system is initially in its state of lowest energy  $\epsilon_1$ , the corresponding wave function being  $\psi_1$ . It is then subjected to a small time-dependent perturbation for a finite interval, after which the wave function will be of the form

$$\psi = \sum c_r \psi_r,$$

where the  $c_r$  are constants, and  $\sum |c_r|^2 = 1$ . Now Einstein argued, "Does  $\psi$  describe a real state of the system? If the answer is yes, then we can hardly do otherwise than to ascribe to this state a definite energy  $\epsilon$ , and, in particular, an energy which exceeds  $\epsilon_1$  by a small amount (in any case  $\epsilon_1 < \epsilon < \epsilon_2$ )." But the experiments on electron impact by J. Franck and G. Hertz strongly indicate that an individual system can only be one of the discrete energies  $\epsilon_1, \epsilon_2, \dots, \epsilon_r$ , etc.

Einstein therefore concluded that  $\psi$  cannot describe a homogeneous state of the system, but rather it must represent a statistical description in which the  $c_r$  are related to the probabilities of the individual discrete energy values.

It seems to be clear, therefore, that Born's statistical interpretation of quantum theory

is the only possible one.<sup>20</sup> The  $\psi$  function does not in any way describe a state which could be that of a single system; it relates rather to many systems, to "an ensemble of systems" in the sense of statistical mechanics. If, except for certain special cases, the  $\psi$  function furnishes only *statistical* data concerning measurable magnitudes, the reason lies not only in the fact that the *operation of measuring* introduces unknown elements, which can be grasped only statistically, but because of the fact that the  $\psi$  function does not, in any sense, describe the state of *one* single system.

He went on to describe how this *statistical ensemble* interpretation satisfactorily resolves the paradox of Einstein, Podolsky and Rosen, which could not be resolved satisfactorily as long as  $\psi$  was assumed to be a complete description of an *individual* system.

Einstein's interpretation of quantum theory is also expressed in a book with Infeld.<sup>21</sup> Because this book was written at the popular level it attracts less attention from physicists than do research papers. However, the subject is expounded with such care and clarity that even the expert can understand it. To explain the statistical nature of quantum theory, the authors first use classical analogies such as population statistics and the kinetic theory of gases, in which no predictions can be made for individual cases but the average behavior in a very large number of cases can be predicted. They then point out that exactly the same is true of typical quantum problems such as radioactive decay and electron scattering. But having pointed out the similarity of the classical and quantum problems, they stress the important difference. The statistical knowledge of birth rates

...is gained by the knowledge of individual cases. Similarly, in the kinetic theory of matter, we have statistical laws governing the aggregation, gained on the basis of individual laws.

But in quantum physics the state of affairs is entirely different. Here the statistical laws are given immediately... Quantum physics abandons individual laws of elementary particles and states *directly* the statistical laws governing aggregations.

One should bear in mind that during these and later years Einstein's major research activities were not concerned with quantum theory, but with general relativity and with attempts to formulate a unified field theory. The latter program takes general relativity as its starting point, and attempts to describe electromagnetism and gravitation as parts of one total field.<sup>22</sup> It was hoped that the sources of this field, charge and mass, would not remain as entities distinct from the field, but that they would be described as concentrated, highly nonlinear regions of the field. One could hope that such a fundamental theory of matter might lead to a better understanding of quantum phenomena.

So one might have expected Einstein to have been pleased when Schrödinger wrote,<sup>23</sup> "I have really believed for a long time that the  $\psi$ -waves are to be identified with waves representing disturbances of the gravitational potential." But in his reply Einstein did not express any interest in such an idea. Instead he wrote,<sup>24</sup> "I am as convinced as ever that the wave representation of matter is an incomplete representation of the state of affairs... The prettiest way to show this is by your example with the cat."

#### Schrödinger's Cat

The above quotation refers to an argument introduced by Schrödinger in a review article published in 1935.<sup>25</sup> Suppose a cat is placed in a chamber together with a machine which is triggered by the decay of a single radioactive atom to trip a hammer and break a bottle of cyanide. If one describes this entire system according to quantum theory, then after a time duration equal to one half-life of the radioactive substance, the state vector of the system will be a linear combination of equal parts of state vectors for a live cat and a dead cat.

This argument effectively refutes certain otherwise plausible interpretations of quantum mechanics. Someone who maintained that the state vector provided a *complete* description of the *individual* system could maintain that an electron in a state  $\psi(\mathbf{r})$  simply possessed *no* definite position, but that it was potentially present at all points for which  $\psi(\mathbf{r})$  was nonzero. That interpretation was plausible because the uncertainty in position is usually confined to atomic dimen-

sions. But in this case the microscopic uncertainty is transformed into a macroscopic uncertainty in the position of the hammer, and leads to the absurd proposition that the cat is neither alive nor dead.

Earlier it could have been maintained that the emergence of a definite result when the position of an electron was measured did not contradict the assertion that the electron previously had no definite position, but rather this was a *transition from the possible to the actual* (reduction of the wave function) caused by the disturbance due to the act of measurement. But in this case the "measurement" consists only in looking to see whether the cat is alive or dead, and no one will believe that the act of looking killed the cat.

Schrödinger's argument supports the conclusion reached by Einstein, Podolsky, and Rosen in the same year. Moreover, it does so without using the concept of *elements of reality*, which drew criticism to the EPR argument.

#### MISUNDERSTANDINGS AND CLARIFICATIONS

Just as Einstein's true attitude toward quantum theory can be determined from his writings, so the origin of the misconceptions and myths about him can be found in writing *about* Einstein by his contemporaries. One of the most influential of these was Max Born. In the final chapter of his *Natural Philosophy of Cause and Chance* Born wrote,<sup>26</sup> "As I have mentioned before Einstein does not accept it [the statistical interpretation of quantum mechanics], but still believes in and works on a return to a deterministic theory."

We have already seen that the first part of this statement is not correct, and we shall later see that the emphasis on determinism was not appropriate. In support of his statement, Born quoted Einstein's more-or-less joking remarks about God playing dice, but he did not consider Einstein's substantial arguments.

Born sent Einstein a preprint copy of this chapter ("Metaphysical Conclusions") and Einstein returned it with several marginal comments, some of which are reproduced in BEL (letter No. 86, dated 18 March 1948). Einstein objected that the meaning of his quotations had been distorted, and at the end of the article he remarked, "The whole thing is rather sloppily thought out,

and for this I must respectfully clip your ear." (The reader should not imagine that these remarks arose from personal animosity. In fact, Born and Einstein remained close friends in spite of disagreements and misunderstandings about quantum mechanics.)

In an attempt to clarify his misunderstood ideas, Einstein sent Born a short essay entitled "Quantum Mechanics and Reality".<sup>27</sup> In it he contrasted two possible interpretations:

(a) The [free] particle really has a definite position and a definite momentum, even if they cannot both be ascertained by measurement in the same individual case. According to this point of view, the  $\psi$ -function represents an incomplete description of the real state of affairs.

(b) In reality the particle has neither a definite momentum nor a definite position; the description by  $\psi$ -function is in principle a complete description.

He then repeated the EPR argument that interpretation (b) is not consistent with the principle that an operation performed in a region of space *A* should not influence the condition of an object in some distant region *B*. Unfortunately the argument was only summarized in general terms without any specific example, and Born apparently missed the point of it.<sup>28</sup>

In 1949 the volume *Albert Einstein: Philosopher-Scientist*<sup>3</sup> was published, containing Einstein's autobiography, essays by scientists and philosophers on various aspects of his work, and Einstein's reply to these essays. Several physicists expressed disapproval of Einstein's attitude toward quantum theory, some of them (Pauli, Born, and Heitler) without even mentioning Einstein's specific arguments! Bohr's essay, "Discussion with Einstein," is the best attempt to deal seriously with Einstein's arguments, but it is mostly concerned with thought experiments attempting simultaneous measurements of canonically conjugate variables, a problem of only peripheral relevance to quantum theory and to Einstein's interpretation of it.

In his "Reply to Criticisms," Einstein used the argument of Schrödinger's cat paradox, although he humanely omitted the cat. Instead he con-

sidered a radioactive atom and a Geiger-counter connected to a chart-recorder which records on paper the moment of decay of the atom. Just as the quantal state vector for an isolated atom will not describe the *actual* moment of decay but only the *probability* of each possible decay time, so the state vector for this system of atom plus apparatus will not describe a definite mark on the recorder chart. Rather, the state vector of the system will be a linear combination (coherent superposition) of components, each of which corresponds to a mark on a different spot on the chart. If one adhered to the view that the state vector provided a complete description of the individual system, one would be forced to the conclusion that the mark on the paper has no objective existence (i.e. independent of any subsequent observation which could be said to "reduce" the wave function). So he concluded,

The attempt to conceive the quantum-theoretical description as the complete description of the individual systems leads to unnatural theoretical interpretations, which become immediately unnecessary if one accepts the interpretation that the description refers to ensembles of systems and not to individual systems.

In his first letter following the publication of the above volume, Born expressed agreement with Einstein's ensemble interpretation.<sup>29</sup>

To say that  $\psi$  describes the "state" of one single system is just a figure of speech, just as one might say in every day life: "My life expectancy (at 67) is 4.3 years!"... what it really means, of course, that you take all individuals of 67 and count the percentage of those who live for a certain length of time. This has always been my own concept of how to interpret  $|\psi|^2$ .

Although this statement helped to resolve their disagreement, Einstein stressed in his reply that they still diverged in their attitudes toward this (now admitted) "incompleteness" of quantum-theoretical descriptions: i.e., whether it was a defect to be removed, or just a fact to be placidly accepted.

There was to be one more misunderstanding between Born and Einstein. When Born retired

from his Chair at the University of Edinburgh, he was presented with a volume of essays by several of his fellow scientists.<sup>30</sup> Of his own contribution, Einstein said, "It is meant to demonstrate the indispensability of your statistical interpretation of quantum mechanics, which Schrödinger, too has recently tried to avoid."<sup>31</sup>

In his essay he considered a free particle with specified energy  $E = \hbar\omega$  moving between two reflecting walls. The appropriate wave function,

$$\psi = A \exp(-i\omega t) \cos kx,$$

yields equal probabilities for the two values of momentum,  $\hbar k$  or  $-\hbar k$ , and a position probability distribution extending uniformly (except for an oscillatory fine structure) from one wall to the other. This remains true even in the *classical limit* ( $\hbar \rightarrow 0$  or mass  $m \rightarrow \infty$ , with  $E$  fixed). In this limit the quantum mechanical description does not go over into the classical description of a body whose center of mass is well defined and oscillating between the walls. However the limit of the description given by  $\psi$  is appropriate to an *ensemble* of such bodies, such as might be considered in classical statistical mechanics. Therefore Einstein concluded, once again, that in general  $\psi$  cannot be regarded as a complete description of an individual system, but rather it describes a statistical ensemble.

To this argument Born objected that one should take  $\psi$  to be a wave packet, rather than an energy eigenstate, before passing to the macroscopic limit ( $m \rightarrow \infty$ ). Einstein insisted that if quantum theory is correct then the classical description should be recovered in this limit for *all*  $\psi$  which satisfy Schrödinger's equation, not just for the restricted class which Born wished to consider, and he emphasized that no difficulty occurs if one adopts the ensemble interpretation.

This discussion continued inconclusively in several letters, until Pauli (who was visiting in Princeton) took it upon himself to mediate the dispute. After examining the arguments of both parties, he wrote to Born<sup>32</sup> that in Einstein's opinion and his own, Born had been a poor listener during the debate. "It seems to me as if you had erected some dummy Einstein for yourself, which you then knocked down with great pomp," wrote Pauli. He then gave a concise



logical summary of Einstein's argument, and stressed that, contrary to Born's misinterpretation, the concept of *determinism* played no role in it. Rather it was the principle of *realism* upon which Einstein had insisted. (That is, a macroscopic body must always have a quasi-sharp position of its center of mass. In Schrödinger's cat paradox the postulate of realism takes the form that the condition of the cat, alive or dead, is an objective fact independent of any observation process.) Although Pauli endorsed all the aspects of Einstein's argument to which Born had objected, he withheld his support from the conclusion because of some doubt (which he only partly made clear) about the principle of realism.<sup>33</sup>

### CONCLUDING REMARKS

We have seen ample evidence that, contrary to the statement by Heisenberg quoted at the beginning of this article, Einstein did advance a definite counterproposal to what is loosely called the Copenhagen interpretation of quantum mechanics, and that far from rejecting the Born's statistical interpretation, he insisted upon it. But in his own interpretation Einstein went even further in this direction by insisting that a quantum state function  $\psi$  must be considered to describe an *ensemble* of similar systems, and cannot legitimately be regarded as a *complete* description of an *individual* system.

Because the predictions of quantum mechanics are statistical, it is necessary to consider an ensemble of repetitions a measurement (e.g., a measurement of position) in order to test the theory. Thus a person who held the second (individual) interpretation would not reject the predictions of the first (ensemble) interpretation, and it is only by analyzing suitable examples (e.g., the EPR argument, Schrödinger's example with the cat, or Einstein's discussion of the classical limit) that the weakness of the second interpretation becomes apparent.

It is important to understand the sense in which Einstein regarded quantum mechanics as an *incomplete* description of physical reality. It is *not* incomplete *merely* because it is a statistical theory and so does not predict individual events. Rather it is considered incomplete because the state function, in general, does not even *describe* the individual event/system. For example, in the

EPR argument it is concluded (from principles which were never challenged *before* the argument was published) that a free particle may have both a definite position and a definite momentum, which no wave function can describe. In Schrödinger's argument, the Schrödinger equation produces a state function that, if it is regarded as referring to one individual system, does not describe the (presumed) fact that the cat is either alive or dead independently of the act of observation. These arguments involve a certain philosophical element (as do all physical theories, if one is honest), but *determinism* is not assumed. For example, the classical theory of Brownian motion is indeterministic and yields only statistical predictions, but it does contain a complete *description* of its object (the position of the Brownian particle as a function of time).

It is sometimes suggested that Einstein's view was akin to so-called hidden-variable theories, but that is an over simplification. When the first example of a hidden-variable theory was published,<sup>34</sup> Einstein wrote to Born,<sup>35</sup> "Have you noticed that Bohm believes (as deBroglie did, by the way, 25 years ago) that he is able to interpret the quantum theory in deterministic terms? That way seems too cheap to me."

Einstein believed that the statistical quantum theory, like thermodynamics, was correct (within the domain of applicability of its concepts) and would necessarily be included in any future theory of greater generality. But, for the reasons already discussed, he did not believe that quantum mechanics could serve as a *starting point* for a unified basis of all physics. He did regard the general theory of relativity as a suitable starting point, and he made it the basis of his unified field theory program. Bohm's hidden variable theory did not establish any deeper connections between quantum mechanics and electromagnetism, gravitation, or any other theory. Thus for Einstein it was "too cheap" to take seriously.

No nontrivial combination of general relativity with quantum theory has ever been accomplished. The unified field theory program has not been able to encompass quantum phenomena, and the other extreme, "quantization" of gravitation by forcing the equations of general relativity into the operator form of quantum mechanics, has also met severe difficulties. Whether or not Einstein's

final research project, the nontrivial unification of physical theory, can be accomplished remains an open question.

<sup>1</sup> G. J. Whitrow, *Einstein, The Man and His Achievement* (British Broadcasting Corporation, London, 1967).

<sup>2</sup> W. Heisenberg, in *Niels Bohr and the Development of Physics*, ed. by W. Pauli (Pergamon, Oxford, 1955).

<sup>3</sup> *Albert Einstein: Philosopher-Scientist*, ed. by P. A. Schilpp (Library of the Living Philosophers, Evanston, Ill., 1949; reprinted by Harper and Row, New York, 1959).

<sup>4</sup> M. Born, *The Born-Einstein Letters* (Walter and Company, New York, 1971).

<sup>5</sup> *Letters on Wave Mechanics*, ed. by K. Przibram, transl. by M. J. Klein (Philosophical Library, New York, 1967). Letters between Schrödinger and Planck, Einstein, and Lorentz.

<sup>6</sup> N. Boni, M. Russ, and D. H. Lawrence, *A Bibliographical Checklist and Index to the Published Writings of Albert Einstein* (Pageant Books, Paterson, N. J., 1960).

<sup>7</sup> LWS, letter No. 12, dated 26 April 1926.

<sup>8</sup> E. Schrödinger, *Ann. der Physik* **79**, 734 (1926).

<sup>9</sup> BEL, letter No. 50.

<sup>10</sup> BEL, letter No. 52.

<sup>11</sup> *Electrons et Photons*, Institut International de Physique Solvay, Rapports et Discussions du Cinquième Conseil de Physique (Gauthier-Villars, Paris, 1928), pp. 253-6.

<sup>12</sup> I have translated the quotations from the original French.

<sup>13</sup> L. E. Ballentine, *Rev. Mod. Phys.* **42**, 358 (1970).

<sup>14</sup> N. Bohr, "Discussion with Einstein on the Epistemological Problems in Atomic Physics," in Ref. 3.

<sup>15</sup> J. L. Park and H. Margenau, *Int. J. Theor. Phys.* **1**, 211 (1968). See also Sec. 3 of Ref. 13. The validity of the above statement is, however, relative to a definition of "measurement." Plausible definitions of this term can be given, according to which "simultaneous measurement" would not be possible [W. E. Lamb, Jr., *Phys. Today* **22**, No. 4, 23 (1969)]. Neither Bohr nor Einstein were very precise about this definition.

<sup>16</sup> E. Prugovečki has attempted to extend the formalism of quantum theory to deal with such measurements. See *Can. J. Phys.* **45**, 2173 (1967).

<sup>17</sup> A. Einstein, B. Podolsky, and N. Rosen, *Phys. Rev.* **47**, 777 (1935).

<sup>18</sup> Sec. 2 of Ref. 13.

<sup>19</sup> A. Einstein, *J. Franklin Institute* **221**, 313 (in German), 349 (in English) (1936).

<sup>20</sup> This quotation, which is reaffirmed in latter years, should lay to rest the myth that Einstein rejected the statistical interpretation of quantum theory.

<sup>21</sup> A. Einstein and L. Infeld, *The Evolution of Physics* (Simon and Schuster, New York, 1938), pp. 280-94.

<sup>22</sup> The discovery of strong and weak interactions has convinced most people that such attempts were premature.

<sup>23</sup> LWM, letter No. 15, dated 19 July 1939.

<sup>24</sup> LWM, letter No. 16, dated 9 August 1939.

<sup>25</sup> E. Schrödinger, *Naturwiss.* **23**, 807 (1935).

<sup>26</sup> M. Born, *Natural Philosophy of Cause and Chance* (Oxford U. P., Oxford, 1949), 2nd ed., Dover, New York (1964).

<sup>27</sup> A. Einstein, *Dialectica* **2**, 320 (1948). An English translation is included in BEL following letter no. 88, dated 5 April 1948.

<sup>28</sup> In the commentary following his reply (BEL, letter No. 89, dated 9 May 1948) Born confuses the possibility that a measurement of *A* can give information about the condition of an object at *B* with impossibility (according to Einstein) of such a measurement influencing or disturbing the condition of the object at *B*. It is curious that no mention of the EPR argument was made in the Born-Einstein correspondence following its publication in 1935. Born's letter in 1948 gives no indication that he could have been familiar with the argument.

<sup>29</sup> BEL, letter No. 96, postscript dated 4 September 1950.

<sup>30</sup> *Scientific Papers Presented to Max Born* (Oliver and Boyd, Edinburgh/London, 1953).

<sup>31</sup> BEL, letter No. 103, dated 12 October 1953.

<sup>32</sup> BEL, letter No. 115, dated 31 March 1954.

<sup>33</sup> See Ref. 32, p. 223. Because there seems to be no fundamental difference between micro- and macro-bodies, it should in principle be possible for even a macro-body to undergo diffraction scattering. Pauli believed this to be inconsistent with the existence of a quasi-sharp position for the body. Actually this objection is not valid, for diffraction-like scattering of quantum mechanical particles is, in general, due to the quantization of momentum transfer to and from a periodic scatterer (see Ref. 13, p. 362). An extended wave function need not (and does not, in Einstein's interpretation) mean an extended particle. Pauli then said, "The appearance of a definite position  $x_0$  during a subsequent observation [after diffraction scattering]... and the statement 'the particle is there', is then regarded as being a 'creation' existing outside the laws of nature, even though it cannot be influenced by the observer." In view of this metaphysical monstrosity, Pauli's later statement that Einstein had 'got stuck in his metaphysics' (BEL, letter No. 116) is worse than the pot calling the kettle black!

<sup>34</sup> D. Bohm, *Phys. Rev.* **85**, 166, 180 (1952).

<sup>35</sup> BEL, letter No. 99, dated 12 May 1952.