

Chapter 21



Entanglement

Entanglement is a mysterious quantum phenomenon that is widely, but mistakenly, described as capable of transmitting information over vast distances faster than the speed of light. It has proved very popular with science writers, philosophers of science, and many scientists who hope to use the mystery to deny one or more of the basic concepts underlying quantum physics.

Some commentators say that *nonlocality* and *entanglement* are a “second revolution” in quantum mechanics, “the greatest mystery in physics,” or “science’s strangest phenomenon,” and that quantum physics has been “reborn.” They usually quote ERWIN SCHRÖDINGER as saying

“I would not call that one but rather the characteristic trait of quantum mechanics,” the one that enforces its entire departure from classical lines of thought.”¹

SCHRÖDINGER knew that his two-particle wave function could not have the same simple interpretation as the single particle, which can be visualized in ordinary three-dimensional configuration space. And he is right that entanglement exhibits a richer form of the “action-at-a-distance” and nonlocality that ALBERT EINSTEIN had already identified in the collapse of the single particle wave function.

The main difference is that two particles instead of one acquire new properties, and they do it instantaneously (at faster than light speeds), just as in the case of a single-particle measurement, where the finite probability of appearing at various distant locations collapses to zero at the instant the particle is found somewhere. This two-particle instantaneous interaction is *nonseparability*.

We can disagree with Schrödinger, who was enthusiastic about the Einstein-Posolsky-Rosen attack in 1935 on quantum mechanics as “incomplete” and who gave the phenomenon the name “entanglement.” In fact, the entanglement of two indistinguishable

¹ *Mathematical Proceedings of the Cambridge Philosophical Society*, Volume 31, Issue 04, October 1935, pp 555-563



particles can be completely understood with PAUL DIRAC's *principle of superposition*, his *axiom of measurement*, and his *projection postulate*. These three fundamentals of quantum mechanics already explain the "mysterious" phenomena that are impossible in classical mechanics, notably the one-particle mystery in the two-slit experiment that RICHARD FEYNMAN calls "the only mystery" in quantum mechanics.

Information philosophy analyzes both the single-particle and two-particle wave function "collapses" as a question of who knows what when, that is, what information exists at each moment and where about the particle(s).

Entanglement depends on two quantum properties that are simply impossible in "classical" physics. One is called *nonlocality*. The other is *nonseparability*. Each of these might be considered a mystery in its own right, but fortunately *information physics* (and the information interpretation of quantum mechanics) can explain them both, with no equations, in a way that should be understandable to the lay person.

This may not be good news for the science writers and publishers who turn out so many titles each year claiming that quantum physics implies that there are multiple parallel universes, that the minds of physicists are manipulating "quantum reality," that there is nothing "really" there until we look at it, that we can travel backwards in time, that things can be in two places at the same time, that we can teleport material from one place to another, and of course that we can send signals faster than the speed of light.

A second concern for Einstein was that the wave function ψ for an isolated free particle evolves in time to occupy all space. All positions become equally probable. Yet when we observe the particle, it is always located at some particular place. This does not prove that the particle had a particular place before the observation, but Einstein had a commitment to "elements of reality" that he thought no one could doubt. One of those elements is a particle's position. He asked the question, "Does the particle have a precise position the moment before it is measured?" The Copenhagen answer was sometimes "no," more often it was "we don't know," or "Don't ask?"



Einstein's Discovery of Nonlocality and Nonseparability

ALBERT EINSTEIN was the first to see the nonlocal character of quantum phenomena. He may have seen it as early as 1905, the same year he published his special theory of relativity. But it was perfectly clear to him twenty-two years later (ten years after his general theory of relativity and his explanation of how quanta of light are emitted and absorbed by atoms), when he described nonlocality to a conference of physicists from around the world in Belgium in 1927 at the fifth Solvay conference.

Then a few years later, in 1935, Einstein, BORIS PODOLSKY, and NATHAN ROSEN proposed a thought experiment (known by their initials as EPR) to exhibit what they thought were internal contradictions in the new quantum physics. Einstein hoped to show that quantum theory could not describe certain intuitive “elements of reality” and thus was either *incomplete* or, as he hoped, demonstrably incorrect.

Einstein and his colleagues Schrödinger, MAX PLANCK, DAVID BOHM, and others hoped for a return to deterministic physics, and the elimination of mysterious quantum phenomena like *superposition of states* and the “*collapse*” of the wave function. EPR continues to fascinate determinist philosophers of science who hope to prove that quantum indeterminacy does not exist.

Beyond the problem of nonlocality, the EPR “thought experiment” introduced the problem of “*nonseparability*.” This mysterious phenomenon appears to transfer something physical faster than the speed of light. Actually there is merely an instantaneous change in the *immaterial* information about probabilities or possibilities for locating the particles.

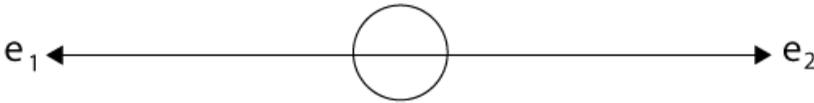
The 1935 EPR paper was based on a question of Einstein's about two electrons fired in opposite directions from a central source with equal velocities. He imagined them starting at time t_0 some distance apart and approaching one another with high velocities. Then for a short time interval from t_1 to $t_1 + \Delta t$ the particles are in contact with one another.



After the particles are measured at t_1 , quantum mechanics describes them with a single two-particle wave function that is not separable into the product of two independent single-particle wave functions. Because electrons are *indistinguishable* particles, it is not proper to say electron 1 goes this way and electron 2 that way. (Nevertheless, it is convenient to label the particles, as we do in the illustrations below.) Until the next measurement, it is misleading to think that specific particles have distinguishable paths. Either particle could be anywhere.

Einstein said correctly that at a later time t_2 , measurement of one electron's position would instantly establish the position of the other electron - without measuring it explicitly.

Electrons are prepared with opposite linear momenta
and travel apart from the center.



If the position of electron 1 is measured at some time, the position of electron 2 must be exactly opposite by conservation of linear momentum. So measuring one tells you something about the other at a great distance. Apparently information travels faster than light speed.

Figure 21-19. Einstein's first explanation of "action-at-a-distance."

In this first discussion of the problem, Einstein simply used *conservation of linear momentum* to calculate the position of the second electron. Although conservation laws are rarely cited as the explanation, they are the reason that entangled particles always produce correlated results. If the results were not always correlated, the implied violation of a fundamental conservation law would be a much bigger story than entanglement itself, as interesting as that is.

Although Einstein mentioned conservation in the original EPR paper, it is noticeably absent from later work. An exception is EUGENE WIGNER, writing on the problem of measurement in 1963:

If a measurement of the momentum of one of the particles is carried out — the possibility of this is never questioned — and gives the result p , the state vector of the other particle suddenly becomes a (slightly damped) plane wave with the momentum $-p$. This statement is synonymous with the statement that a measurement of the momentum of the second par-



ticle would give the result $-p$, as follows from the conservation law for linear momentum.²

This idea of something measured in one place “influencing” measurements far away challenged what Einstein thought of as “local reality.” He famously called nonseparability “*spukhafte Fernwirkungen*” or “spooky action at a distance.” But there is no action here. It might better be called “knowledge at a distance.”

Einstein had objected to nonlocal phenomena as early as the Solvay Conference of 1927, when he criticized the collapse of the wave function as “instantaneous-action-at-a-distance.”

Oddly, Einstein’s criticism resembles the criticisms by RENÉ DESCARTES and others about Newton’s theory of gravitation. Newton’s opponents charged that his theory was “action at a distance” and instantaneous. Einstein’s general relativity shows that gravity is not instantaneous. It travels at the speed of light and is mediated by a gravitational field that can be viewed mathematically as curvature in space-time.

But note that when a probability function collapses to unity in one place and zero elsewhere, nothing physical is moving from one place to the other.

In 1964, JOHN BELL showed how the 1935 “thought experiments” of Einstein, Podolsky, and Rosen (EPR) could be made into real physical experiments. Bell put limits on the “hidden variables” that might deny *nonlocality* and possibly restore a deterministic physics. His test was in the form of what he called *inequalities*, the violation of which would confirm standard quantum mechanics.

Since Bell’s work, many other physicists have defined other “Bell inequalities” and developed increasingly sophisticated experiments to test them. every test confirming standard quantum mechanics.

The first practical and workable experiments to test the EPR paradox had been suggested by DAVID BOHM in 1952. Instead of only linear momentum conservation, Bohm proposed using two electrons that are prepared in an initial state of known total spin. If one electron spin is $1/2$ in the up direction and the other is spin down or $-1/2$, the total spin is zero. The underlying physical law of importance is a second conservation law, in this case the conservation of

2 “The Problem of Measurement,” in *Quantum Theory and Measurement*, Wheeler and Zurek, p.340



angular momentum. If electron 1 is prepared with spin down and electron 2 with spin up, the total angular momentum is zero.

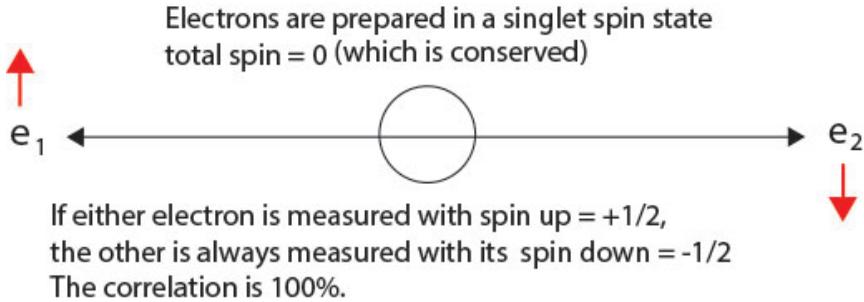


Figure 21-20. David Bohm changed EPR to measure electron spins.

Quantum theory says the two electrons are in a *superposition* of combined spin up (+) and spin down (-) states,

$$|\psi\rangle = 1/\sqrt{2} |+-\rangle + 1/\sqrt{2} |-+\rangle \quad (1)$$

The principles of quantum mechanics say that the prepared system is in a linear combination of these two states, and can provide only the probabilities of finding the entangled system in either the $|+-\rangle$ state or the $|-+\rangle$ state. Quantum mechanics does not describe the paths or the spins of the individual particles. Note that should measurements result in a $|++\rangle$ or $|--\rangle$ state, that would violate the conservation of angular momentum.

EPR tests can be done more easily with polarized photons than with electrons, which require complex magnetic fields. The first of these was done in 1972 by STUART FREEDMAN and JOHN CLAUSER at UC Berkeley. They used oppositely polarized photons (one with spin = +1, the other spin = -1) coming from a central source. Again, the total photon spin of zero is conserved. Their data, in agreement with quantum mechanics, violated Bell's inequalities to high statistical accuracy, thus providing strong evidence against local hidden-variable theories and confirming quantum mechanics.

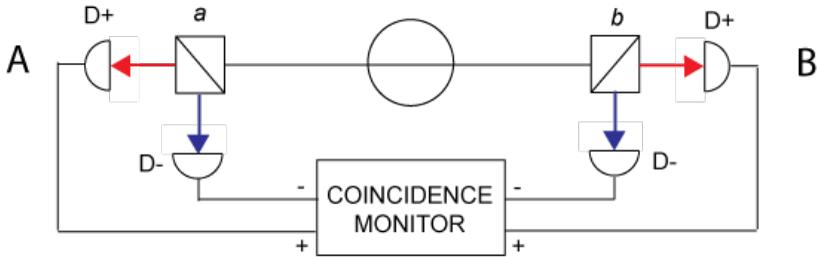
For more on superposition of states and the physics of photons, see the Dirac 3-polarizers experiment in appendix B.

Clauser, MICHAEL HORNE, ABNER SHIMONY, and RICHARD HOLT (known collectively as CHSH) and later ALAIN ASPECT did more sophisticated tests. The outputs of the polarization analyzers were



fed to a coincidence detector that records the instantaneous measurements, described as $+ -$, $- +$, $++$, and $--$. The first two ($+ -$ and $- +$) conserve the spin angular momentum and are the only types ever observed in these nonlocality/entanglement tests.

A typical CHSH apparatus sends the signals from the polarization analyzers to a coincidence monitor.



The coincidence monitor then counts four kinds of events, N_{++} , N_{+-} , N_{-+} , and N_{--} . Perfect correlation (and conservation of spin angular momentum) allows only $+ -$ and $- +$ events.

Figure 21-21. The CHSH teams looked for perfect synchronization.

With the exception of some of Holt's early results that were later found to be erroneous, *no evidence has so far been found of any failure of standard quantum mechanics*. And as experimental accuracy has improved by orders of magnitude, quantum physics has correspondingly been confirmed to one part in 10^{16} , and the speed of any transfer of information between particles has a lower limit of 10^6 times the speed of light. There has been no evidence for local "hidden variables."

NICOLAS Gisin and his colleagues have extended the polarized photon tests of EPR and the Bell inequalities to a separation of 18 kilometers near Geneva. They continue to find 100% correlation and no evidence of the "hidden variables" sought after by Einstein and DAVID BOHM.

Nevertheless, wishful-thinking experimenters continue to look for possible "loopholes" in the experimental results, such as detector inefficiencies that might be hiding results favorable to Einstein's picture of "local reality."



The Importance of Conservation Laws in Entanglement

Conservation laws are the consequence of extremely deep properties of nature that arise from simple considerations of symmetry. We regard these laws as “cosmological principles.” Physical laws do not depend on the absolute place and time of experiments, nor their particular direction in space. Conservation of linear momentum depends on the translation invariance of physical systems, conservation of energy the independence of time, and conservation of angular momentum the invariance under rotations.

Recall that the EPR experiment starts with two electrons (or photons) prepared in an entangled state that is a linear combination of pure two-particle states, each of which conserves the total angular momentum and, of course, conserves the linear momentum as in Einstein’s original EPR example. The initial information about the linear and angular momenta is established by the state preparation (a measurement).

Quantum mechanics describes the probability amplitude wave function ψ of the two-particle system as in a superposition of two-particle states. It is not separable into a product of single-particle states, and there is no information about the identical indistinguishable electrons traveling along distinguishable paths.

The probability amplitude wave function ψ travels from the source (at the speed of light or less). Let’s assume that at t_1 observer A finds an electron (e_1) with spin up.

After the “first” measurement, new information comes into existence telling us that the wave function ψ has “collapsed” into the state $| + - \rangle$. Just as in the two-slit experiment, probabilities have now become certainties. If the “first” measurement finds electron 1 is spin up, so the entangled electron 2 must be found by observer B to be in a “second” measurement with spin down to conserve angular momentum.

Notice that Einstein’s intuition is in part correct that the “second” result seems already “determined” or “fixed” before the second measurement. The result is determined by the law of conservation of momentum that the total the spin must remain zero.



But the measurement by observer B was not pre-determined before observer A's measurement. It was simply *determined by her measurement*. And conservation of linear momentum tells us that at t_1 the second electron is equidistant from the source in the opposite direction.

As with any wave-function collapse, the probability amplitude information “travels” instantly.

But unlike the single particle in the two-slit experiment, where the collapse goes to a specific point in 3-dimensional configuration space, the “collapse” here is a “projection” into one of the two possible 6-dimensional two-particle quantum states $| + - \rangle$ or $| - + \rangle$.

Just as the single particle in the two-slit experiment did not have a position before the measurement, the two particles, which just before the measurement did not have positions, instantly acquire their positions in a space-like separation after the measurement.

This makes “visualization” (Schrödinger's *Anschaulichkeit*) more difficult, but the parallel with the collapse in the two-slit case provides an intuitive insight of sorts.

Schrödinger said that his “Wave Mechanics” provided more “*visualizability*” than the “damned quantum jumps” of the Copenhagen school, as he called them. He was right.

But we must focus on the probability amplitude wave function of the prepared two-particle state, and not attempt to describe the paths or locations of independent particles - which is only possible after some measurement has been made. We must also keep in mind the conservation laws that Einstein used to describe nonlocal behavior in the first place. Then we can see that the “mystery” of nonlocality for two particles is primarily the same mystery as the single-particle collapse of the wave function. But there is an extra mystery, one we might call an “enigma,” of the *nonseparability* of identical indistinguishable particles.

In his 1935 paper (and his correspondence with Einstein), Schrödinger described the two particles in EPR as “entangled” in English, *verschränkt* in German, which means something like cross-linked. It describes someone standing with arms crossed.



In the time evolution of an entangled two-particle state according to the Schrödinger equation, we can visualize it (just as we visualize the single-particle wave function) as collapsing when a measurement is made. The discontinuous “jump” is also described as the “reduction of the wave packet.” This is apt in the two-particle case, where the superposition of $| + - \rangle$ and $| - + \rangle$ states is “projected” or “reduced to one of these states, say $| - + \rangle$, and then further reduced to the product of two independent one-particle states, $| - \rangle | + \rangle$.

Measurement of a two-particle wave function simultaneously measures both particles, reducing them to *separate* one-particle wave functions, after which they are no longer entangled.

When entangled, the particles are *nonseparable*. Once measured, they are separate quantum systems with their own wave functions. They are no longer entangled.

In the two-particle case (instead of just one particle making an appearance), when either particle is measured we know instantly the now determinate properties of the other particle. They are the properties that satisfy the conservation laws, including its location equidistant from, but on the opposite side of, the source, and the complementary spin.

In the one-particle case, it has no definite position before the experiment, then it appears somewhere. For two particles, neither one has a position, then both appear simultaneously (in an appropriate frame of reference and with required opposite spins).³

Can a Special Frame Resolve the EPR Paradox?

Almost every presentation of the EPR paradox begins with something like “Alice observes one particle...” and concludes with the question “How does the second particle get the information needed so that Bob’s measurements correlate perfectly with Alice?”

There is a fundamental *asymmetry* in this framing of the EPR experiment. It is a surprise that Einstein, who was so good at seeing deep symmetries, did not consider how to remove the asymmetry.

3 For an animation of a two-particle measurement, see informationphilosopher.com/solutions/experiments/EPR/EPR-collapse.gif



Consider this reframing: Alice's measurement collapses the two-particle wave function. The two indistinguishable particles simultaneously appear at locations in a space-like separation. The frame of reference in which the source of the two entangled particles and the two experimenters are at rest is a *special frame* in the following sense.

As Einstein knew very well, there are frames of reference moving with respect to the laboratory frame of the two observers in which the time order of the events can be reversed. In some moving frames Alice measures first, but in others Bob measures first.

If there is a special frame of reference (not a *preferred frame* in the relativistic sense), surely it is the one in which the origin of the two entangled particles is at rest. Assuming that Alice and Bob are also at rest in this special frame and equidistant from the origin, we arrive at the simple picture in which any measurement that causes the two-particle wave function to collapse makes both particles appear simultaneously at determinate places with fully correlated properties (just those that are needed to conserve energy, momentum, angular momentum, and spin).

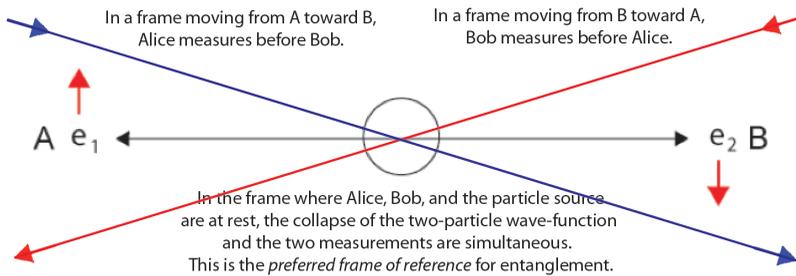


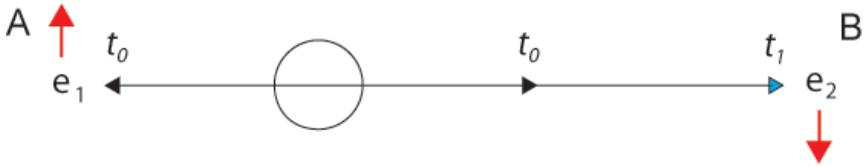
Figure 21-22. In this special frame the source and measurements are at rest and both measurements are made at exactly the same time.

Clearly, the idea that different relativistic frames of reference change the order of the measurements throws doubt on claims by either observer to “measure first.”

We can also ask what happens if Bob is not at the same distance from the origin as Alice. This introduces a positional asymmetry. But there is still no time asymmetry from the point of view of the two-particle wave function collapse.



If observer A measures electron 1 with spin up at t_0 ,
electron 2 *instantly* is spin down (Ψ collapses to $|+ - \rangle$).



Electron 2 is then *determined* to be found with spin down
if measured by observer B at a later time t_1 .

But this was not *predetermined* before A's measurement at t_0 .

Figure 21-23. Here Alice measures long before Bob's measurement.

When Alice detects a particle (with spin up), at that instant the other particle also becomes determinate (with spin down) at the same distance on the other side of the origin. It continues, in that determinate state, to Bob's measuring apparatus.

Our idea of a special frame is not new.

Back in the 1960's, C. W. RIETDIJK and HILARY PUTNAM independently, but mistakenly, argued that physical determinism could be proved true by considering the experiments and observers A and B in the diagram below to be moving at high speed with respect to one another. ROGER PENROSE developed a similar argument in his book *The Emperor's New Mind*, called the "Andromeda Paradox."

Chapter 21

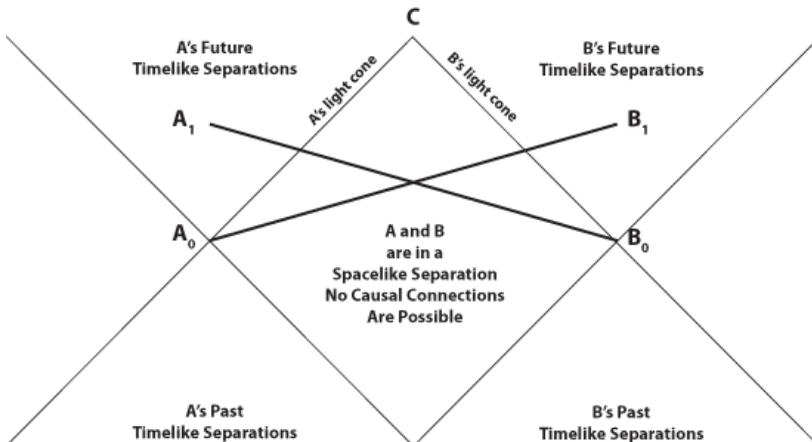


Figure 21-24. Physicists have known about our "special frame" for decades.



NICOLAS GISIN's colleagues, ANTOINE SUAREZ and VALERIO SCARANI, used this idea of hyperplanes of simultaneity to make what they called “before-before” measurements.

Suarez and Scarani used the fact that for some relative speeds between the two observers A and B, observer A could “see” the measurement of observer B to be in his future, and vice versa. Because the two experiments have a “space-like” separation (neither is inside the causal light cone of the other), *each observer thinks he does his own measurement before the other.*

Gisin tested the limits on this effect by moving mirrors in the path to the birefringent crystals and showed that, like all other Bell experiments, the “before-before” suggestion of Suarez and Scarani did nothing to invalidate quantum mechanics.

But these experiments were able to put a lower limit on the speed with which the information about probabilities collapses, estimating it as at least thousands - perhaps millions - of times the speed of light and showed empirically that probability collapses are essentially instantaneous.

Despite all his experimental tests verifying quantum physics, including the “reality” of nonlocality and entanglement, Nicolas Gisin continues to explore the EPR paradox, considering the possibility that signals are coming to the entangled particles from “outside space-time.”

Do We Need Superdeterminism?

During a mid-1980's interview by BBC Radio 3 organized by P. C. W. DAVIES and J. R. BROWN, John Bell proposed the idea of a “*superdeterminism*” that could explain the correlation of results in entangled two-particle experiments without the need for faster-than-light signaling. The two experiments need only have been pre-determined by causes reaching both experiments from an earlier time.

Davies: I was going to ask whether it is still possible to maintain, in the light of experimental experience, the idea of a deterministic universe?

Bell: You know, one of the ways of understanding this business is to say that the world is super-deterministic. That not only is inanimate nature



deterministic, but we, the experimenters who imagine we can choose to do one experiment rather than another, are also determined. If so, the difficulty which this experimental result creates disappears.⁴

Bell's superdeterminism would deny the important "free choice" of the experimenter (originally suggested by Bohr and Heisenberg) and later explored by JOHN CONWAY and SIMON KOCHEN. Conway and Kochen claim that the experimenters' free choice requires that atoms must have free will, something they call their *Free Will Theorem*.

In his 1996 book, *Time's Arrow and Archimedes' Point*, HUW PRICE proposes an Archimedean point "outside space and time" as a solution to the problem of nonlocality in the Bell experiments in the form of an "advanced action."

Rather than a "superdeterministic" common cause coming from "outside space and time" (as proposed by Bell, Gisin, Suarez, and others), Price argues that there might be a cause coming backwards in time from some interaction in the future. Penrose and STUART HAMEROFF have also promoted this idea of "backward causation," sending information backward in time in the Libet experiments and in the EPR experiments.

EPR "Loopholes" and Free Will

Investigators who try to recover the "elements of local reality" that Einstein wanted, and who hope to eliminate the irreducible randomness of quantum mechanics that follows from wave functions as probability amplitudes, often cite "loopholes" in EPR experiments. For example, the "detection loophole" claims that the efficiency of detectors is so low that they are missing many events that might prove Einstein was right.

Most all the loopholes have now been closed, but there is one loophole that can never be closed because of its metaphysical/philosophical nature. That is the "(pre-)determinism loophole."

4 *The Ghost in the Atom*, P.C.W. Davies and J. Brown, ch.3, p.47



If every event occurs for reasons that were established at the beginning of the universe, then all the careful experimental results are meaningless. Conway and Kochen have formalized this loop-hole in what they call the Free Will Theorem.

Although Conway and Kochen do not claim to have proven free will in humans, they assert that should such a freedom exist, then the same freedom must apply to the elementary particles.

What Conway and Kochen are really describing is nothing more than the indeterminism that quantum mechanics has introduced into the world. Although indeterminism is a requirement for human freedom, it is insufficient by itself to provide both “free” and “will” as we saw in chapter 4.

We also need the *adequate or statistical determinism* in the second stage of “free will” to ensure that whatever our “free choice” may be, it has been made consistent with our reasons for the choice.

There are no such considerations of reasons, motives, feelings, etc. going on at the quantum level for electrons. But Conway and Kochen are right about the fundamental connection between quantum indeterminism and free will.

