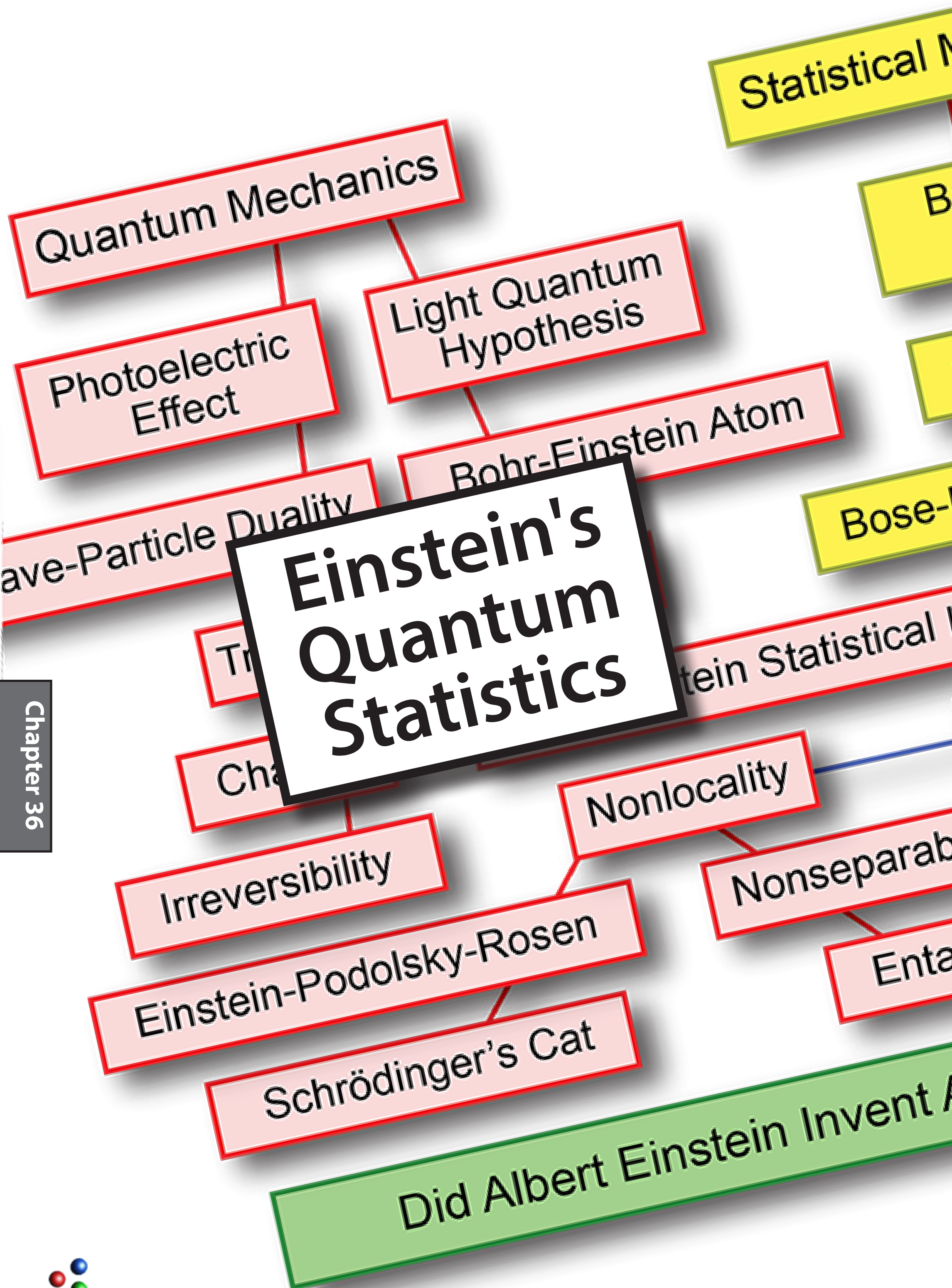
Einstein's Quantum Statistics
Quantum Mechanics
Photoelectric Effect
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Did Albert Einstein Invent

Chapter 36

Einstein's Quantum Statistics

We saw in chapter 5 that Einstein rederived all of classical statistical mechanics between 1902 and 1904, going beyond the kinetic theory of gases developed by LUDWIG BOLTZMANN in the nineteenth century. Twenty years later, Einstein discovered quantum statistics. Prompted by a new derivation of Planck's radiation distribution law by SATYENDRA NATH BOSE, Einstein showed that the distribution of photons differs from Boltzmann's molecular distribution by the addition of a -1 in the denominator.

Shortly after Einstein's paper, PAUL DIRAC showed that fermions (spin 1/2 particles) also depart from the Boltzmann distribution, by the addition of a +1 in the denominator.

No. of (atoms/molecules) $\sim (1 / (e^{E/kT}))$.

No. of (bosons) $\sim (1 / (e^{E/kT} - 1))$.

No. of (fermions) $\sim (1 / (e^{E/kT} + 1))$.

Einstein's quantum statistics gave us the first examples of the two fundamental kinds of particle in the standard model of particle physics - fermions and bosons. See chapter 15.

All of this *before* the "founders" of quantum mechanics discovered the equations that allow us to *calculate* quantum properties to extraordinary levels of accuracy.

Einstein did not care much for the details of calculation, except to prove a fundamental theory. Just two years later, after WERNER HEISENBERG had developed matrix mechanics and ERWIN SCHRÖDINGER created wave mechanics, Einstein generously allowed his friend MAX BORN to take full credit for the "statistical interpretation" of quantum mechanics, which Einstein had seen qualitatively well over a decade earlier (chapter 20).

To be sure, Born identified Einstein's *qualitative* probability with the calculated squared modulus of Schrödinger's wave function $|\psi|^2$. This made the statistical interpretation *quantitative*.

As we have seen so well, Einstein was very unhappy about the ontological implications of the statistics he discovered. He said many times to Born over the next few decades, "God does not play dice," But over those decades Born never noticed that Einstein had embraced *indeterminism* in quantum mechanics . Einstein's criticisms were mostly directed to *nonlocality* (chapter 23).

Elementary Particles Are Not Independent

In 1927 Einstein worried that his quantum statistics were telling him that the indistinguishability and interchangeability of elementary particles may mean that they are *not independent* of one another. Is this part of the reason particles are entangled, even when they are apparently at a great distance from one another?

If the two-particle wave function Ψ_{12} does not factor into products of single-particle functions $\Psi_1$and Ψ_2, *i*t is telling us that the two particles are not independent of one another.

Einstein wrote Schrödinger in 1925.

> In the Bose statistics employed by me, the quanta or molecules are not treated as being *independent of one another.* [1]

In 1927, Einstein asked whether Schrödinger's wave mechanics determine a system *completely*, or only *statistically*. This was the fundamental issue of his EPR paper eight years later. It was a question of whether $\Psi_{12} \Rightarrow \Psi_1\Psi_2$

> a system Σ is considered, which consists of two energetically mutually independent partial systems Σ_1 and Σ_2. [If the first] contains only quantities with reference to Σ_1, the second, only quantities with reference to Σ_2, Then as is known, $\Psi = \Psi_1\Psi_2$ .... The indicated schema does not correspond to this condition. In particular, let μ be an index belonging to a coordinate of Σ_1, ν an index belonging to a coordinate of Σ_2. Then $\Psi_{\mu\nu}$ does not vanish. [2]

When Einstein published EPR in 1935, Schrödinger wrote him approvingly within weeks, but he followed up with two papers on "probability between separated systems," in which he pointed out the same disturbing facts of nonseparability that Einstein had wrestled with since he saw their interdependence in his quantum statistics nine years earlier. The collapse of his two-particle wave function results in "entangled" particles, as he first called them.

1 February 28, 1925, CPAE vol. 14, doc. 446,
2 May 3, 1927, CPAE vol. 15, doc. 516.

Einstein referred obliquely to two entangled particles in a 1948 paper. He laments their conflict with his idea of a field theory.

> physical things ... are conceived of as being arranged in a space-time continuum... [they] claim an existence independent of one another, insofar as these things "lie in different parts of space." [One does not] see how physical laws could be formulated and tested without such a clean separation. Field theory has carried out this principle to the extreme, in that it localizes within infinitely small (four-dimensional) space elements.[3]

Einstein apparently considered the entanglement of electron *spins* as suggested by David Bohm in 1952. One of the editors of the *Collected Papers of Albert Einstein* found a manuscript in which Einstein wrote these few short lines:

> Composite system of total spin 0.
>
> 1) The description is assumed to be complete.
>
> 2) A coupling of distant things is excluded.
>
> If the spin of the subsystem I is measured along the x-axis, it is found to be either 1 or −1 in that direction.
>
> But if instead the spin of subsystem I is measured along the y-direction, it follows that the spin of the subsystem II is equal to 1 or −1.
>
> If there is no coupling, then the result of a measurement of the spin of subsystem II may in no way depend on whether a measurement was taken of subsystem I (or on what kind of measurement).
>
> The two assumptions therefore cannot be combined.
>
> If the description is not assumed to be complete for the individual system, then that what is being described is not a single system but an ensemble of systems. Then a measurement of subsystem I amounts to the selection of a subensemble of the ensemble of the total system. Then the prediction for a measurement of subsystem II can depend on the choice of the measurement of subsystem I.

Then these two lines were in the right margin:

> a) the description by the quantum theory is an incomplete one with respect to the individual system, or
>
> b) there is an immediate coupling of states of spatially separated things.[4]

So much for the *impossibility of simultaneity*?

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3 Einstein,1948, p.322.

4 Sauer, 2007, p.884