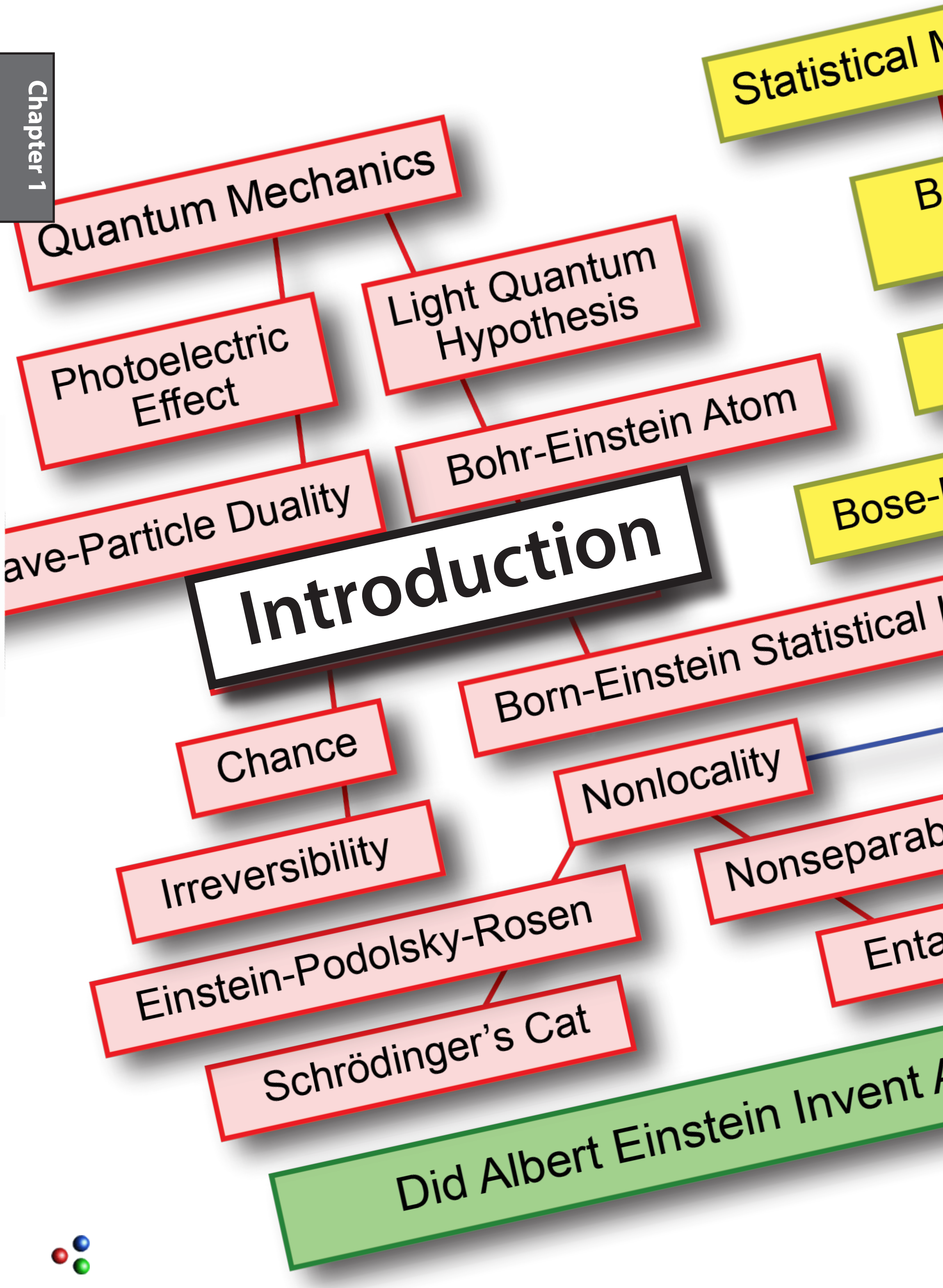
Chapter 1



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Did Albert Einstein Invent

Introduction

This book is the story of how Albert Einstein analyzed what goes on when *light* interacts with *matter* and how he discovered ontological *chance* in the process. We can show that Einstein's chance explains the metaphysical possibilities underlying the creation of all of the *information structures* in the universe.

But the story begins with a deck of cards, a pair of dice, and the multiple flips of a coin.

Around 1700, Abraham de Moivre, a French Huguenot, emigrated to England to escape religious persecution. A brilliant mathematician, he worked with Isaac Newton and other great English scientists, but he could never get an academic post, despite their excellent recommendations. To support himself, de Moivre wrote a handbook for gamblers called *The Doctrine of Chances*.

This was not the first book that calculated the odds for different hands of cards or rolls of the dice. But when de Moivre considered the flipping of a fair coin (with 50-50 odds of coming up heads and tails) he showed that as the number of flips gets large, the *discrete* binomial distribution of outcomes approaches a *continuous* curve we call the Gaussian distribution (after the great mathematician Carl Friedrich Gauss), the "normal" distribution, or just the "bell curve," from its familiar shape.

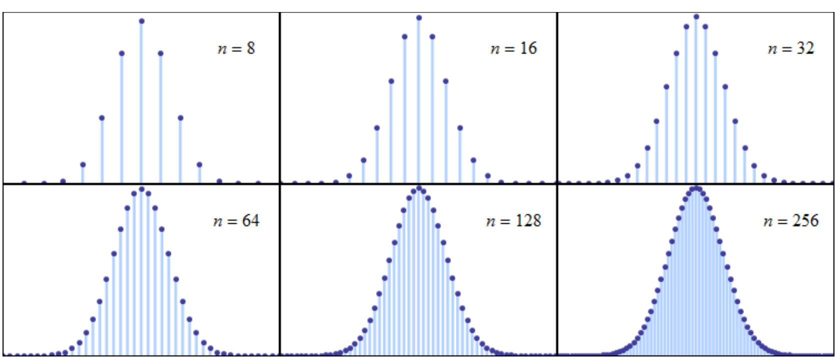


Figure 1-1. De Moivre's discovery of the continuous bell curve as a limit to a large number of discrete, discontinuous events. Each discrete event is the probability of m heads and n-m tails in n coin tosses. The height is the coefficient in the binomial expansion of $(p + q)^n$ where $p = q = ½$.

In mathematics, we can say that a finite number of discrete points approaches a continuum as we let the number approach infinity. This is the "law of large numbers" and the "central limit theorem."

But in physics, the continuous appearance of material things is only because the discrete atoms that make it up are too small to see. The analytic perfection of the Gaussian curve cannot be realized by any finite number of events.

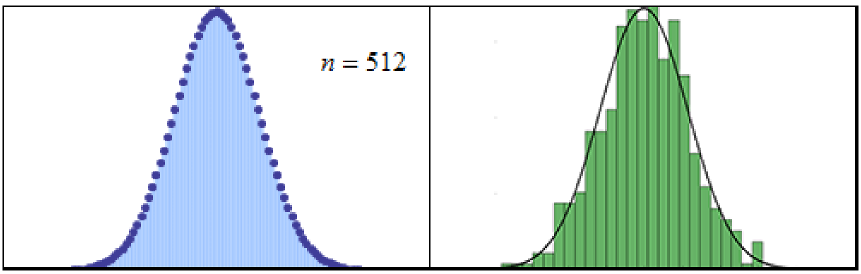


Figure 1-2. The appearance of a continuous curve and actual finite events.

Is the Nature of Reality Continuous or Discrete?

Is it possible that the physical world is made up of nothing but discrete discontinuous *particles*? Are continuous *fields* with well-defined values for matter and energy at all places and times simply theoretical constructs, averages over large numbers of particles?

Space and time themselves have well-defined values everywhere, but are these just the abstract information of the *ideal* coordinate system that allows us to keep track of the positions and motions of particles? Space and time are physical, but they are not *material*.

We use material things, rulers and clocks, to measure space and time. We use the abstract mathematics of real numbers and assume there are an *infinite number* of real points on any line segment and an infinite number of moments in any time interval. But are these continuous functions of space and time nothing but *immaterial* ideas with no material substance?

The two great physical theories at the end of the nineteenth century, Isaac Newton's classical mechanics and James Clerk Maxwell's electrodynamics, are *continuous field theories*.

Solutions of their field equations determine precisely the exact forces on any material particle, providing complete information about their past and future motions and positions. Field theories are generally regarded as *deterministic* and *certain*.

Although the dynamical laws are "free inventions of the human mind," as Einstein always said,[1] and although they ultimately depend on experimental evidence, which is always *statistical*, the field theories have been considered superior to merely statistical laws. Dynamical laws are thought to be *absolute*, based on *principles*.

We will find that the continuous, deterministic, and analytical laws of classical dynamics and electromagnetism, expressible as differential equations, are idealizations that "go beyond experience."

These continuous laws are to the discontinuous and discrete particles of matter and electricity (whose motions they describe perfectly) as the analytical normal distribution above is to the finite numbers of heads and tails. A *continuum* is approached in the limit of large numbers of particles, when the random fluctuations of individual events can be averaged over.

Experiments that support physical laws are always finite in number. Experimental evidence is always *statistical*. It always contains *errors* distributed randomly around the most probable result. And the distribution of those errors is often *normal*.

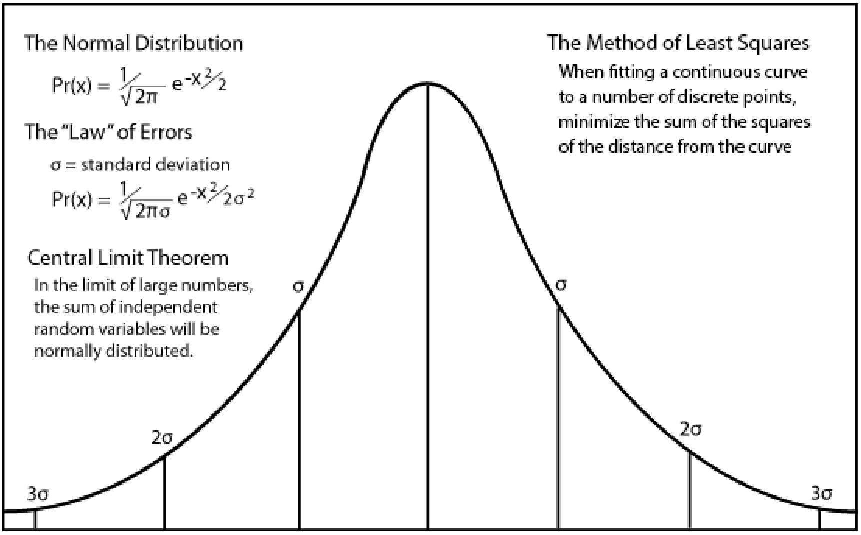


Figure 1-3. Random errors are normally distributed around the mean value.

1 EInstein, 1934, p.234

The Absolute Principles of Physics

There are of course *absolute* principles in physics, such as the conservation laws for mass/energy, momentum, angular momentum, and electron spin. The constant velocity of light is another.

The great mathematician EMMY NOETHER proposed a theorem that conservation principles are the consequence of deep *symmetry* principles of nature. She said for any property of a physical system that is symmetric, there is a corresponding conservation law.

Noether's theorem allows physicists to gain insights into any general theory in physics, by analyzing the various transformations that would make the form of the laws involved *invariant*.

For example, if a physical system is symmetric under rotations, its angular momentum is conserved. If it is symmetric in space, its momentum is conserved. If it is symmetric in time, its energy is conserved. Now locally there is time symmetry, but cosmically the expansion of the universe gives us an arrow of time connected to the increase of entropy and the second law of thermodynamics.

The conservation of energy was the *first law* of thermodynamics.

The famous *second law* says entropy rises to a maximum at thermal equilibrium. It was thought by most scientists to be an absolute law, but we shall see in chapter 3 that Maxwell and LUDWIG BOLTZMANN considered it a statistical law. Boltzmann thought it possible that a system that had reached equilibrium might spontaneously back away, if only temporarily, from the maximum. Assuming that the universe had an infinite time to reach equilibrium, he thought it might be that the non-equilibrium state we find ourselves in might be a giant *fluctuation*. Given his assumption of infinite time, even such an extremely improbable situation is at least *possible*.

In his early work on statistical mechanics, Einstein showed that small *fluctuations* in the motions of gas particles are constantly leading to departures from equilibrium. Somewhat like the departures from the smooth analytic bell curve for any finite number of events, the entropy does not rise smoothly to a maximum and then stay there indefinitely. The second law is not *continuous* and *absolute*.

The second law of thermodynamics is unique among the laws of physics because of its *irreversible* behavior. Heat flows from hot into cold places until they come to the same equilibrium temperature. The one-direction nature of *macroscopic* thermodynamics (with its gross "phenomenological" variables temperature, energy, entropy) is in fundamental conflict with the assumption that *microscopic* collisions between molecules, whether fast-moving or slow, are governed by dynamical, deterministic laws that are time-reversible. But is this correct?

The microscopic second law suggests the "arrow of time" does not apply to the time-reversible dynamical laws. At the atomic and molecular level, there appears to be no arrow of time, but we will see that Einstein's work shows particle collisions are not reversible

The first statistical "laws" grew out of examples in which there are very large numbers of entities. Large numbers make it impractical to know much about the individuals, but we can say a lot about *averages* and the probable distribution of values around the averages.

Probability, Entropy, and Information

Many scientists and philosophers of science say that the concept of entropy is confusing and difficult to understand, let alone explain. Nevertheless, with the help of our diagrams demonstrating probability as the *number of ways* things have happened or been arranged, divided by the total number of ways they might have happened or been arranged, we can offer a brief and visual picture of entropy and its important connection to information.

We begin with Ludwig Boltzmann's definition of the entropy S in terms of the number of ways W that gas particles can be distributed among the cells of "phase space," the product of ordinary coordinate space and a momentum space.

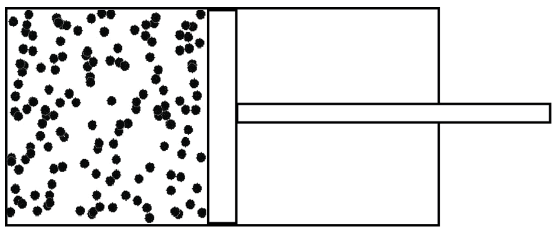
$S = k \log W$

Let's greatly simplify our space by imagining just two cubicle bins separated by a movable piston. Classical thermodynamics was developed studying steam engines with such pistons.

Now let's imagine that a thousand molecules are dropped *randomly* into the two bins. In this very artificial case, imagine that they all land up on the left side of the piston. Assuming the probabilities of falling into the left or right bin are equal, this is again the binomial expansion with $(p + q)^{1000}$ with p = q = ½. All molecules on the left would have probability $(1/2)^{1000}$. This is of course absurdly improbable if each events were random, but steam engines do this all the time, and calculating the improbability gives us a measure of the machine's available energy.

Figure 1-4. An ideal piston with gas on the left and a perfect vacuum on the right.

To see how this very improbable situation corresponds to very low entropy, how low entropy corresponds to maximum information,



and how low entropy means energy available to do work, let's consider the number of yes/no questions needed to figure out the chessboard square where a single pawn is located.

1) Is it in the top half? No.
   Of the remaining half,
2) is it in the left half? No.
   Of the remaining half,
3) Is it in the right half? No.
   Of the remaining half,
4) Is it in the top half? Yes.
   Of the remaining half,
5) Is it in the left half? Yes.
   Of the remaining half,
6) Is it in the top half? Yes.

In Claude Shannon's 1948 theory of the communication of information, the answer to a yes/no question communicates one bit (a binary digit can be 1 or 0) of information. So, as we see, it takes

6 bits of information to communicate the particular location of the pawn on one of the 64 possible squares on the chessboard.

Shannon and his mentor, the great mathematical physicist John von Neumann, noticed that the information *I* is the logarithm of the number of possible ways *W* to position the pawn. Two raised to the 6th power is 64 and the base 2 logarithm of 64 is 6. Thus

$I = log_2 W$ and $6 = log_2 64$

The parallel with Boltzmann's entropy formula is obvious. His formula needs a constant with the physical dimensions of energy divided by temperature (ergs/degree). But Shannon's information has no physical content and does not need Boltzmann's constant *k*. Information is just a dimensionless number.

For Shannon, entropy is the *number of messages* that can be sent through a communications channel in the presence of noise. For Boltzmann, entropy was proportional to the *number of ways* individual gas particles can be distributed between cells in phase space, assuming that all cells are equally probable.

So let's see the similarity in the case of our piston. How many ways can all the 1000 gas particles be found *randomly* on the left side of the piston, compared to all the other ways, for example only 999 on the left, 1 on the right, 998 on the left, 2 on the right, etc.

Out of 2^{1000} ways of distributing them between two bins, there is only *one way* all the particles can be on the left. [2] The logarithm of 1 is zero ($2^0 = 1$). This is the minimum possible entropy and the maximum of available energy to do work pushing on the piston.

Boltzmann calculated the likelihood of random collisions resulting in the *unmixing* of gases, so that noticeably fewer are in the left half of a 1/10 liter container, as of the order of $10^{10^{10}}$ years. [3] Our universe is only of the order of 10^{10} years old.

It seems most unlikely that such chance can lead to the many interesting information structures in the universe. But chance will play a major role in Einstein's description of what he called "objective reality," as we shall see.

2 1000! (factorial) is 1000 x 999 x 998 ... x 2 x 1. (really big)
3 Boltzmann, 2011, *p.444*