VOLUME XL, No. 5 MARCH 4, 1943

THE JOURNAL OF PHILOSOPHY

NOTES ON EXISTENCE AND NECESSITY

THIS paper[1] concerns two points of philosophical controversy. One is the question of admission or exclusion of the modalities—necessity, possibility, and the rest—as operators attaching to statements. The other is the ontological question, "What *is* there?" It is my purpose here to set forth certain considerations, grounded in elementary logic and semantics, which—while not answering either question—must seriously condition any tenable answers.

The logical notions that prove crucial to these considerations are the notions of identity and quantification; and the semantical ones are the notions of designation and meaning, which are insufficiently distinguished in some of the current literature. A new semantical notion that makes its appearance here and plays a conspicuous part is that of the "purely designative occurrence" of a name.

1. Designation and Identity

One of the fundamental principles governing identity is that of *substitutivity*—or, as it might well be called, that of *indiscernibility of identicals.* It provides that, *given a true statement of identity, one of its two terms may be substituted for the other in any true statement and the result will be true.* It is easy to find cases contrary to this principle. For example, the statements:

(1) Giorgione = Barbarelli,

(2) Giorgione was so-called because of his size

are true; however, replacement of the name 'Giorgione' by the name 'Barbarelli' turns (2) into the falsehood:

Barbarelli was so-called because of his size.

Furthermore, the statements:

(3) Cicero = Tully,

(4) 'Cicero' contains six letters

are true, but replacement of the first name by the second turns (4)

[1] Mainly a translation, from the Portuguese, of portions of my forthcoming book *O sentido da nova lógica* (São Paulo, Brazil).

false. Yet the basis of the principle of substitutivity appears quite solid; whatever can be said about the person Cicero (or Giorgione) should be equally true of the person Tully (or Barbarelli), this being the same person.

In the case of (4), this paradox resolves itself immediately. The fact is that (4) is not a statement about the person Cicero, but simply about the word 'Cicero.' The principle of substitutivity should not be extended to contexts in which the name to be supplanted occurs without referring simply to the object.

The relation of name to the object whose name it is, is called *designation;* the name 'Cicero' designates the man Cicero. An occurrence of the name in which the name refers simply to the object designated, I shall call *purely designative.* Failure of substitutivity reveals merely that the occurrence to be supplanted is not purely designative, and that the statement depends not only upon the object but on the form of the name. For it is clear that whatever can be affirmed about the *object* remains true when we refer to the object by any other name.

An expression which consists of another expression between single quotes constitutes a name of that other expression; and it is clear in general that the occurrence of that other expression or any part of it, within the context of quotes, is not designative. In particular the occurrence of the personal name within the context of quotes in (4) is not designative, nor subject to the substitutivity principle. The personal name occurs there merely as a fragment of a longer name which contains, beside this fragment, the two quotation marks. To make a substitution upon a personal name, within such a context, would be no more justifiable than to make a substitution upon the term 'cat' within the context 'cattle'.

The example (2) is a little more subtle, for it is a statement about a man and not merely about his name. It was the man, not his name, that was called so and so because of his size. Nevertheless, the failure of substitutivity shows that the occurrence of the personal name in (2) is not *purely* designative. It is easy in fact to translate (2) into another statement which contains two occurrences of the name, one purely designative and the other not:

(5) Giorgione was called 'Giorgione' because of his size.

The first occurrence is purely designative. Substitution on the basis of (1) converts (5) into another statement equally true:

Barbarelli was called 'Giorgione' because of his size.

The second occurrence of the personal name is no more designative than any other occurrence within a context of quotes.

To get an example of another common type of statement in which names do not occur designatively, consider any person who is called Philip and satisfies the condition:

(6) Philip is unaware that Tully denounced Catiline,

or perhaps the condition:

(7) Philip believes that Tegucigalpa is in Nicaragua.

Substitution on the basis of (3) transforms (6) into the statement:

(8) Philip is unaware that Cicero denounced Catiline,

no doubt false. Substitution on the basis of the true identity:

Tegucigalpa = Capital of Honduras

transforms the truth (7) likewise into the falsehood:

(9) Philip believes that the capital of Honduras is in Nicaragua.

We see, therefore, that the occurrences of the names 'Tully' and 'Tegucigalpa' in (6)–(7) are not purely designative.

In this there is a fundamental contrast between (6), or (7), and:

Crassus heard Tully denounce Catiline.

This statement affirms a relation between three persons, and the persons remain so related independently of the names applied to them. But (6) can not be considered simply as affirming a relation between three persons, nor (7) a relation between person, city, and country—at least, not so long as we interpret our words in such a way as to admit (6) and (7) as true and (8) and (9) as false.

Some readers may wish to construe unawareness and belief as relations between persons and statements, thus writing (6) and (7) in the manner:

Philip is unaware of 'Tully denounced Catiline',

Philip believes 'Tegucigalpa is in Nicaragua',

the purpose being to put within a context of single quotes every not purely designative occurrence of a name. It is not necessary, however, to force an analogy thus between cases of the type (6)–(7) and those of the type (4)–(5). It is unnecessary to insist that every indesignative occurrence of a name form part of the name of an expression. What is important is to insist that the contexts 'is unaware that . . .' and 'believes that . . .' are, like the context of single quotes, contexts in which names do not occur purely designatively. The same is true of the contexts 'knows that . . .', 'says that . . .', 'doubts that . . .', 'is surprised that . . .', etc.

2. Designation and Quantification

We have observed a basic connection between designation and identity. We have next to examine a connection, equally basic, between designation and existence—existence as expressed in the prefix '$\exists x$' of existential quantification in logic.

It must be noted carefully, to begin with, that this prefix has the very broad sense 'there is something x such that', and does not connote existence in any peculiarly spatial or temporal sense. The statement:

$$\exists x(x \text{ is a fish} \cdot x \text{ flies})$$

does affirm the existence of something in space and time, but only because fishes and things that fly are always in space and time, and not because of any spatial sense of '$\exists x$'. The prefix is no less suited to the context:

$$\exists x(x \text{ is a prime number} \cdot x \text{ is between 5 and 11}).$$ [2]

The intimate connection between designation and existential quantification is implicit in the operation of *existential generalization* —the operation whereby, from 'Socrates is mortal', we infer '$\exists x(x$ is mortal)', i.e., 'Something is mortal'. The idea behind such inference is that whatever is true of the object designated by a given substantive is true of something; and clearly the inference loses its justification when the substantive in question does not happen to designate. From:

There is no such thing as Pegasus,

for example, we do not infer:

$$\exists x(\text{there is no such thing as } x),$$

i.e., 'There is something which there is no such thing as', or 'There is something which there is not'.

Inference by existential generalization is of course equally unwarranted in the case of an indesignative occurrence of any substantive, whether of 'Pegasus' (which never occurs designatively) or of 'Giorgione', 'Cicero', 'Tegucigalpa', etc. (which often do occur designatively). Let us see what in fact happens in some of

[2] The special emphasis put by philosophers on the distinction between existence as applied to spatio-temporal objects and existence (or subsistence or being) as applied to abstract objects, or universals, is partly prompted by an idea that the methods of knowing existence in the two cases are basically different. But this idea, according to which the observation of nature is relevant only to determining the existence of spatio-temporal particulars and never the being of universals, is readily refuted by counter-instances such as that of "hyperendemic fever" in my paper "Designation and Existence," this Journal, Vol. XXXVI (1939), p. 703.

these further cases. From (2), existential generalization would lead to:

$$\exists x(x \text{ was so-called because of its size}),$$

i.e., 'Something was so-called because of its size'. This is clearly meaningless, there being no longer any suitable antecedent for 'so-called'. Note, in contrast, that existential generalization with respect to the purely designative occurrence in (5) yields the sound conclusion:

$$\exists x(x \text{ was called 'Giorgione' because of its size}),$$

i.e., 'Something was called 'Giorgione' because of its size'.

Applied to the occurrence of the personal name in (4), existential generalization would lead us to:

(10) $\exists x(\text{'}x\text{'} \text{ contains six letters}),$

i.e.:

(11) There is something such that 'it' contains six letters,

or perhaps:

(12) 'Something' contains six letters.

Any expression formed by single quotes is a name of the expression within the quotes. In particular, thus, the expression:

'x' contains six letters

means simply:

The 24th letter of the alphabet contains six letters.

In (10) the occurrence of the letter within the context of quotes is as irrelevant to the quantifier that precedes it as is the occurrence of the same letter in the context 'six'. (10) consists merely of a falsehood preceded by an irrelevant quantifier. (11) is similar; its part:

'it' contains six letters

is false, and the prefix 'there is something such that' is irrelevant. (12), again, is false—if by 'contains six' we mean 'contains exactly six'.

It is less obvious, and correspondingly more important to recognize, that existential generalization is unwarranted likewise in the case of (6) and (7). Applied to (6), it leads to:

$$\exists x(\text{Philip is unaware that } x \text{ denounced Catiline}),$$

i.e.:

(13) Something is such that Philip is unaware that it denounced Catiline.

What is this object, that denounced Catiline without Philip yet having become aware of the fact? Tully, i.e., Cicero? But to suppose this would conflict with the fact that (8) is false.

Note that (13) is not to be confused with:

Philip is unaware that $\exists x$(x denounced Catiline),

which, though it happens to be false, is quite straightforward and in no danger of being inferred by existential generalization from (6).

The logical operation of *application* is that whereby we infer from 'Everything is itself', for example, or in symbols '$(x)(x = x)$', the conclusion that Socrates = Socrates. This and existential generalization are in fact two aspects of a single principle; for instead of saying that '$(x)(x = x)$' implies 'Socrates = Socrates', we could as well say that the denial 'Socrates ≠ Socrates' implies '$\exists x(x \neq x)$'. The principle embodied in these two operations is the link between quantifications and the singular statements that are related to them as instances. Yet it is a "principle" only by courtesy. It holds only in the case where a substantive designates, and, furthermore, occurs designatively. It is simply the logical content of the idea that a given occurrence is designative.[3]

The ontology which one accepts, or which a given context presupposes, is not revealed by an examination of mere vocabulary; for we know that substantives can be used indesignatively without depriving them of meaning. Use of the word 'Pegasus' does not imply acceptance of Pegasus, nor does the mere use of the signs '9' or '9^{9^9}' imply that there are abstract objects, numbers, such as 9 and 9^{9^9}. It is not the mere use of a substantive, but its designative use, that commits us to the acceptance of an object designated by the substantive.

In order to determine whether a substantive is used designatively in a given context we have to look beyond the substantive and observe the behavior of the pronouns. Ways of using the substantive that do commit one to recognition of the object are embodied in the operations of existential generalization and application. The ontology to which one's use of language commits him comprises simply the objects that he treats as falling with the subject-matter of his quantifiers—within the range of values of his variables.

[3] The principle is, for this reason, anomalous as an adjunct to the purely logical theory of quantification. Hence the theoretical importance of the fact that all substantives, except the variables that serve as pronouns in connection with quantifiers, are dispensable and eliminable by paraphrase. See my *Mathematical Logic*, §27. Such elimination of names does not, of course, eliminate any objects; but the contact between language and object comes to be concentrated in the variable, or pronoun.

3. Meaning and Necessity

To say that two names designate the same object is not to say that they are *synonymous*, that is, that they have the same meaning. To determine the synonymity of two names or other expressions it should be sufficient to understand the expressions; but to determine that two names designate the same object, it is commonly necessary to investigate the world. The names 'Evening Star' and 'Morning Star', for example, are not synonymous, having been applied each to a certain ball of matter according to a different criterion. But it appears from astronomical investigations that it is the same ball, the same planet, in both cases; that is, the names designate the same thing. The identity:

(14) Evening Star = Morning Star

is a truth of astronomy, not following merely from the meanings of the words.

It results equally from astronomical researches, and not merely from the meanings of the words, that the object (the number, or degree of multiplicity) designated by the numeral '9' is the same as that designated by the complex name 'the number of planets'. The identity:

(15) The number of planets = 9

is a truth (so far as we know at the moment) of astronomy. The names the 'number of planets' and '9' are not synonymous; they do not have the same meaning. This fact is emphasized by the possibility, ever present, that (15) be refuted by the discovery of another planet.

Another contrast between designation and meaning is that only certain very definite expressions designate (viz., the names of the objects designated), whereas perhaps all words and other more complex unities capable of figuring in statements have meaning. In particular, substantives such as 'Pegasus' that fail to designate are not without meaning; in fact, it is only with an eye to the meaning of 'Pegasus' that we are able to conclude from a study of zoology that the word does not designate.

It is confusion of meaning and designation that gives rise to the quandary: "If there is no such thing as Pegasus, then there is nothing for 'Pegasus' to mean; but then this word and its contexts, even the context 'Pegasus does not exist', are meaningless." This quandary and its like no doubt have constituted a main motive for admitting, in addition to abstract objects and in addition to the concrete objects in space and time, certain further concrete objects which are more or less like the ones in space and time but are merely possible, not actual. Pegasus is admitted as an object, in this widened domain of concrete objects, but one which lacks merely the special property of actuality. It should be apparent, though, that this extravagant multiplication of entities is a very temporary palliative, for in place of 'Pegasus' we can pick an example not accommodated even by the realm of possible objects—say 'the spinster wife of Pegasus'.

Just what the *meaning* of an expression is—what kind of object—is not yet clear; but it is clear that, given a notion of meaning, we can explain the notion of *synonymity* easily as the relation between expressions that have the same meaning. Conversely also, given the relation of synonymity, it would be easy to derive the notion of meaning in the following way: the meaning of an expression is the class of all the expressions synonymous with it. No doubt this second direction of construction is the more promising one. The relation of synonymity, in turn, calls for a definition or a criterion in psychological and linguistic terms. Such a definition, which up to the present has perhaps never even been sketched, would be a fundamental contribution at once to philology and philosophy.

The relation of synonymity is presupposed, as we have seen, in the notion of meaning, which is used so abundantly in every-day discourse. The notion of synonymity figures implicitly also whenever we use the method of indirect quotations. In indirect quotation we do not insist on a literal repetition of the words of the person quoted, but we insist on a *synonymous* sentence; we require reproduction of the *meaning*. Such synonymity differs even from logical equivalence; and exactly what it is remains unspecified.

The relation of synonymity is presupposed also in the notion, so current in philosophical circles since Kant, of *analytic* statements. It is usual to describe an analytic statement as a statement that is true by virtue of the *meanings* of the words; or as a statement that follows logically from the meanings of the words. Given the notion of synonymity, given also the general notion of truth, and given finally the notion of logical form (perhaps by an enumeration of the logical vocabulary), we can define an analytic statement as any statement which, by putting synonyms for synonyms, is convertible into an instance of a logical form all of whose instances are true. For example, Professor Stevenson's favorite analytic statement, 'No spinster is married', is converted into an instance of the form 'No *A* not *B* is *B*' by putting 'woman not married' for its synonym 'spinster'; and this form 'No *A* not *B* is *B*', which is logical in the sense of preserving only words of the logical vocabulary ('no', 'not', 'is'), is a form all of whose instances are true.

Among the various possible senses of the vague adverb 'necessarily', we can single out one—the sense of *analytic* necessity—according to the following criterion: the result of applying 'necessarily' to a statement is true if, and only if, the original statement is analytic.

(16) Necessarily no spinster is married,

for example, is equivalent to:

(17) 'No spinster is married' is analytic,

and is therefore true. The statement:

(18) 9 is necessarily greater than 7

is equivalent to

(19) '9 > 7' is analytic

and is therefore true (if we recognize the reducibility of arithmetic to logic). The statement:

(20) Necessarily, if there is life on the Evening Star then there is life on the Evening Star

is equivalent to:

(21) 'If there is life on the Evening Star, then there is life on the Evening Star' is analytic

(or, as we could also formulate it:

(22) 'There is life on the Evening Star' implies itself analytically,

if we explain a statement as implying another analytically when the conditional formed from the respective statement is analytic). (20) is then true, since the conditional in question is logically true and therefore analytic.

On the other hand the statements:

(23) The number of planets is necessarily greater than 7,

(24) Necessarily, if there is life on the Evening Star then there is life on the Morning Star

are false, since the statements:

The number of planets is greater than 7,

If there is life on the Evening Star, then there is life on the Morning Star

are true only because of circumstances outside logic.

The prefixes 'possibly' and 'it is impossible that' are definable immediately on the basis of 'necessarily' in the fashion 'not necessarily not' and 'necessarily not'. Thus, for example, (16) can be paraphrased in the manner:

(25) It is impossible that some spinsters be married.

4. Non-Truth-Functional Composition of Statements

The statements (17), (19), (21), and (22) are explicitly statements about statements. They attribute the property of analyticity or the relation of analytic implication to statements, referring to statements by use of their names (constructed with single quotes). On the other hand, (16), (18), (20), and (25) do not refer to other statements by use of their names; they are rather compounds of the statements themselves. The prefixes 'necessarily' and 'it is impossible that' are applied, like the sign of denial, to statements to form others.

The contrast between 'necessarily' and 'is analytic' is exactly analogous to the contrast between '$\sim$' and 'is false'. To write the denial sign before the statement itself in the manner:

$$\sim 9 < 7$$

means the same as to write the words 'is false' after the name of the statement, in the manner:

'$9 < 7$' is false.

In the example (20) we can recognize a complex connective, 'necessarily, if-then'. This connective, like 'if-then' or the dot of conjunction, joins statements to form others.

There is nevertheless a striking difference between the compounds reducible to conjunction and denial on the one hand and the compounds (16), (18), (20), and (25) on the other. These latter are *intensional* compounds, in the sense that the truth-value of the compound is not determined merely by the truth-value of the components.

The statements (17), (19), (21), and (22), besides containing names of statements, are also literally *compounds* of these same statements, the quotation marks being part of an expression applied to the component statement to form the compound. Just as the statements '$\sim 9 > 7$' and (18) are formed from the component statement '$9 > 7$' by the application of '$\sim$' and 'necessarily', we may consider that (19) is formed from the same component by application of two quotation marks and the words 'is analytic'. Similarly for (17), (21), and (22).[4]

[4] Cf. E. V. Huntington, "Note on a recent set of postulates," *Journal of Symbolic Logic*, Vol. 4 (1939), pp. 10–14.

The way in which such statements occur in the "compounds" (17), (19), (21), and (22) is, indeed, rather irregular and accidental. In general, we know that all matter within a context of single quotes is isolated, in an important sense, from the broader context. We know that a name within a context of single quotes does not occur designatively, and that a pronoun within such a context does not succeed in referring to a quantifier anterior to the quotes.

It is in the supposed freedom from these defects that the intensional composition of statements by means of 'necessarily', 'possibly', and 'necessarily if-then', like extensional composition by means of '$\sim$' and '$\cdot$', is thought to constitute composition of statements in a more genuine sense than that which puts the component within quotes. The prefixes 'necessarily' and 'possibly' aspire to such uses as:

If an object necessarily has one or other of two attributes, then it is not possible that it lack both attributes,

that is:

$$(x)(y)(z) \sim (y \text{ and } z \text{ are attributes} \cdot \text{necessarily } x \text{ has } y \text{ or } z \cdot \text{possibly } x \text{ lacks } y \text{ and } z),$$

in which a pronoun within the context 'necessarily . . .' or 'possibly . . .' refers beyond that context.

However, the cited modes of intensional composition of statements are, in fact, subject to the same defects as the context of quotes. For, in view of the fact that a substitution on the basis of the true identity (14) transforms the truth (20) into the falsehood (24), we have to conclude that the terminal occurrence of the name 'Evening Star' in (20) is not purely designative. Equally, in view of the fact that a substitution on the basis of the true identity (15) transforms the truth (18) into the falsehood (23) we conclude that the occurrence of the name '9' in (18) is not purely designative.

It follows that the context 'necessarily . . .', at least in the analytic sense which we are considering, is similar to the context of single quotes and to the contexts 'is unaware that . . .', 'believes that . . .', etc. It does not admit pronouns which refer to quantifiers anterior to the context.[5]

The expression:

$$\text{Necessarily} \sim (x) \sim x > 7,$$

that is, 'Necessarily something is greater than 7', still makes sense, being in fact a true statement; but the expression:

$$\sim (x) \sim x \text{ is necessarily greater than } 7,$$

[5] These circumstances must be carefully considered in any appraisal of a calculus of necessity such, for example, as that of C. I. Lewis.

that is, 'There is something which is necessarily greater than 7', is meaningless. For, would 9, that is, the number of planets, be one of the numbers necessarily greater than 7? But such an affirmation would be at once true in the form (18) and false in the form (23). Similar observations apply to the use of pronouns in connection with the example (20). This resistance to quantification, observed in relation to the context 'necessarily . . .', is encountered equally in connection with the derivative contexts 'possibly . . .' etc.

We see, therefore, that the apparent compounds (16), (18), (20), and (25) are compounds of the contained statements only in the irregular or accidental sense noted in the case of contexts which use quotes. It would be clearer, perhaps, to adhere explicitly to the forms (17), (19), (21), and (22), instead of the alternative forms (16), (18), (20), and (25). These observations apply, naturally, to the prefix 'necessarily' only in the explained sense of analytic necessity; and correspondingly for possibility, impossibility, and the necessary conditional. As for other notions of necessity, possibility, etc., for example, notions of physical necessity or possibility, the first problem would be to formulate the notions clearly and exactly. Afterwards we could investigate whether such notions involve non-designative occurrences of names and hence resist the introduction of pronouns and exterior quantifiers. This question concerns intimately the practical use of language. It concerns, for example, the use of the contrary-to-fact conditional within a quantification; for it is reasonable to suppose that the contrary-to-fact conditional reduces to the form 'necessarily, if p and q' in some sense of necessity. Upon the contrary-to-fact conditional depends in turn, for example, this definition of solubility in water: To say that an object is soluble in water is to say that it would dissolve if it were in water. In discussions of physics, naturally, we need quantifications containing the clause 'x is soluble in water', or the equivalent in words; but, according to the definition suggested, we should then have to admit within quantifications the expression 'if x were in water then x would dissolve', that is, 'necessarily if x is in water then x dissolves'. Yet we do not know whether there is a suitable sense of "necessity" that admits pronouns referring thus to exterior quantifiers.[6]

The effect of these considerations is rather to raise questions than to answer them. The one important result is the recognition that any intensional mode of statement composition, whether based on some notion of "necessity" or, for example, on a notion of

[6] For a theory of "disposition terms," like 'soluble,' see Rudolf Carnap, "Testability and Meaning," *Philosophy of Science*, Vol. 3 (1936), pp. 419–471; Vol. 4 (1937), pp. 1–40.

"probability" (as in Reichenbach's system), must be carefully examined in relation to its susceptibility to quantification. Perhaps the only useful modes of statement composition susceptible to quantification are the extensional ones, reducible to '$\sim$' and '$\cdot$'. Up to now there is no clear example to the contrary. It is known, in particular, that no intensional mode of statement composition is needed in mathematics.

5. Attributes and Classes

The use of general terms, like 'man' or 'blue', or of abstract terms, like 'justice' or '9', does not commit us to recognizing the existence of abstract objects. As is already clear, the question of our ontological presuppositions rests rather on our designative use of such terms, and depends finally on our manner of using pronouns and quantifiers. In fact, the question of ontological presuppositions reduces completely to the question of the domain of objects covered by the quantifier.

It turns out, nevertheless, that mathematics depends on the recognition of abstract objects—such as numbers, functions, relations, classes, attributes. The abstract objects upon whose recognition mathematics depends are, in fact, reducible to a part which includes only classes or attributes.[7] But abstract objects, these or others, have to be admitted in the domain of the quantifier.

The nominalist, admitting only concrete objects, must either regard classical mathematics as discredited, or, at best, consider it a machine which is useful despite the fact that it uses ideograms of the form of statements which involve a fictitious ontology. However, anyone who cares to explore the foundations of mathematics must, whatever his private ontological dogma, begin with a provisional tolerance of classes or attributes. But what is the difference between classes and attributes? It is common to speak of a class as a "mere aggregate", and to imagine it as having its members inside it, according to a spatial analogy; whereas an attribute tends to be imagined rather on the analogy of a power that inheres in the object that has the attribute, or as a feature that the object exhibits. This appeal to opposing analogies is pointless. Classes are as abstract and non-spatial as attributes, as I have emphasized elsewhere,[8] and there is no difference between classes and attributes beyond perhaps this: classes are the same when their members are the same, whereas attributes may be regarded as distinct even though possessed by the same objects.

The opinion is sometimes held that the idea of attribute (or property) is more intuitive than that of class, and that the idea of

[7] Cf. by *Mathematical Logic*, Chapters III–VI.

[8] *Op. cit.*, p. 120.

class should be derived from that of attribute. The derivation presents little difficulty,[9] but the idea that such a derivation is desirable is very curious. It rests perhaps on a confusion between attribute and *matrix*, this latter being an expression which has the form of a statement but contains a free variable. Certainly, in order to specify a class we usually have to present a matrix that is satisfied by the members of the class and by them only; but in this respect classes and attributes are alike, for the determination of an attribute also depends, usually, on presenting a matrix satisfied by the objects, and only those that have the attribute. The matrix is not the attribute.

Classes, being abstract objects, are less clear and familiar than we might wish, but attributes are even more obscure; for the only difference between classes and attributes resides, as we have seen, in the condition of identity, and in this respect classes are much clearer than attributes. Two matrices determine the same class when satisfied by the same objects; but under what condition do the matrices determine the same attribute?

Usually no criterion is offered. The only one I know is the following: matrices determine the same attributes if, and only if, they are logically equivalent. But this criterion leads to awkward results. Consider the attributes determined by the respective matrices:

(26) $x >$ number of planets,

(27) $x > 9$;

that is, the attribute of exceeding the number of planets and the attribute of exceeding 9. Since (26) and (27) are not logically equivalent, it follows that the attributes will not be identical. The statement:

(28) The attribute of exceeding the number of planets = the attribute of exceeding 9

is false. Still, substitution in the true statement:

The attribute of exceeding 9 = the attribute of exceeding 9

on the basis of (15) leads to (28). We have to conclude that the occurrence of '9' in the context 'the attribute of exceeding 9' is not purely designative. Likewise, more generally, we must conclude that the occurrences of names within names of attributes are not designative. Expressions of the type that specify attributes are not contexts accessible to pronouns referring to anterior quantifiers.

[9] Cf. Whitehead and Russell, *Principia Mathematica*, vol. 1, *20; also my essay "Whitehead and the rise of modern logic," in *The Philosophy of A. N. Whitehead* (Library of Living Philosophers, 1941), pp. 147 f.

Clearly this constitutes a fundamental restriction on the use of attributes. It is, in particular, a restriction which makes attributes inadequate to the ends of mathematics and inadequate even as a basis for the subsequent introduction of classes. The only recourse would be to adopt another standard for identity of attributes not based on logical equivalence. But what might such an alternative standard be? And would attributes so construed still be as intuitive as classes?

There may still be a reason to maintain that certain attributes are more intuitive than classes—namely, the attributes, properties, or qualities of sense experience, for example, those of color and sound. It is possible to maintain that these attributes are sometimes distinct even though possessed by the same objects, and still to maintain that the difficulty noted in the case of the matrices (26) and (27) does not arise, since (26) and (27) are not among the matrices to which the simple attributes of sense experience correspond. However, such a domain of special attributes, not corresponding to matrices in general, would not suffice for the purposes of mathematics, nor for the derivation of a general theory of classes.

The main conclusions reached in the five sections of this paper are as follows. A substantive word or phrase which designates an object may occur purely designatively in some contexts and not purely designatively in others. This second type of context, though no less "correct" than the first, is not subject to the law of substitutivity of identity nor to the laws of application and existential generalization. Moreover, no pronoun (or variable of quantification) within a context of this second type can refer back to an antecedent (or quantifier) prior to that context. This circumstance imposes serious restrictions, commonly unheeded, upon the significant use of modal operators, as well as challenging that philosophy of mathematics which assumes as basic a theory of attributes in a sense distinct from classes.

WILLARD V. QUINE.

HARVARD UNIVERSITY.

A TECHNIQUE OF PROBLEM SOLUTION

THE technique of the solution of a problem transcends in importance detailed information of a field of knowledge. Problems related to imposed tasks yield to one of the three modes of solution: the experimental method, the method of models, and the analytic method.

The experimental method implies familiarity with similar systems. An extrapolation, within reasonable limits, of experiences