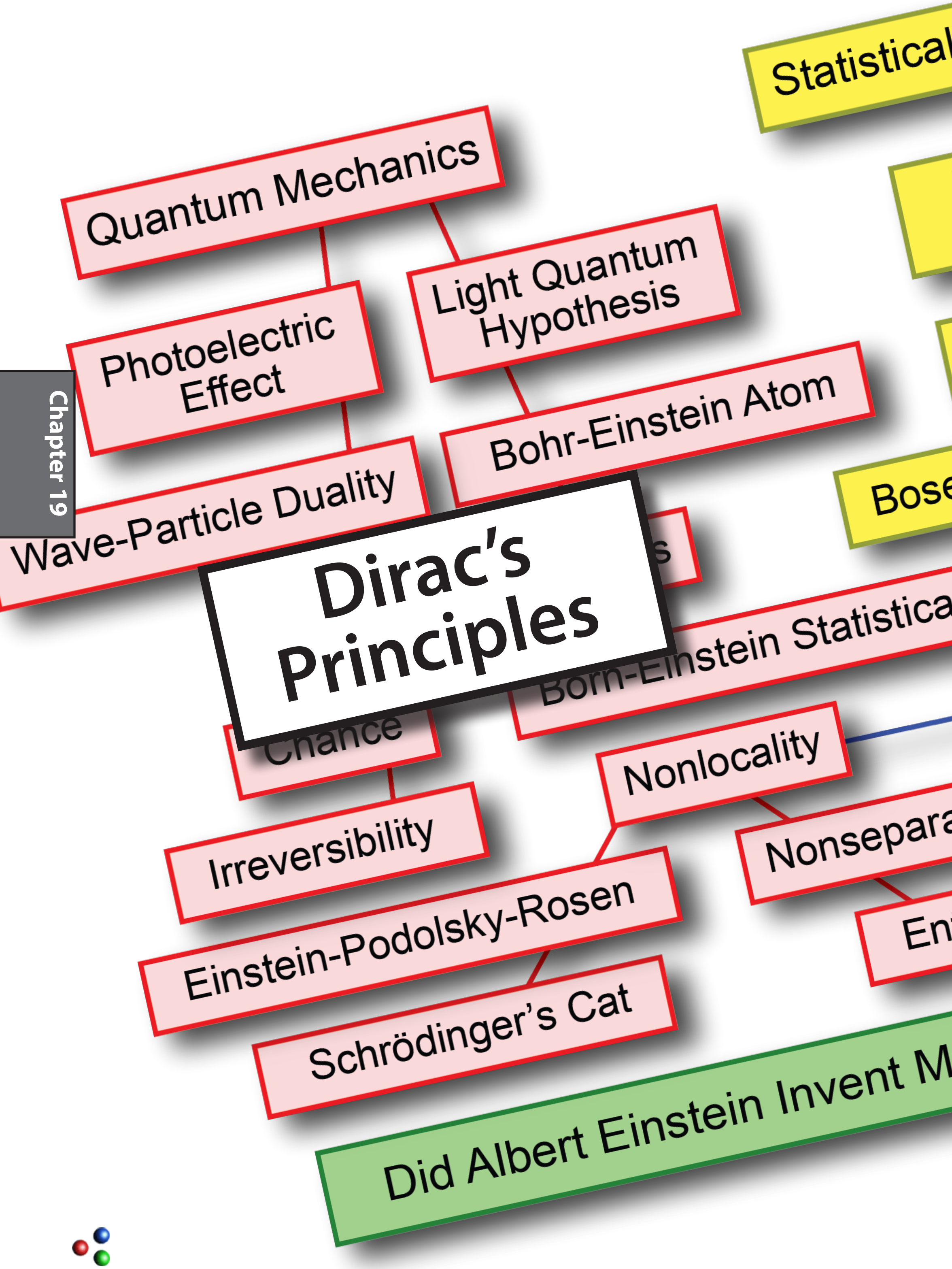
Statistical
Quantum Mechanics
Light Quantum Hypothesis
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Dirac's Principles
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Dirac's Principles of Quantum Mechanics

In 1926 Paul (P.A.M.) Dirac combined the matrix mechanics of Werner Heisenberg and the wave mechanics of Erwin Schrödinger into his beautifully symmetric transformation theory of quantum mechanics.

A year earlier, Dirac had been given a copy of Heisenberg's first paper on quantum mechanics. Heisenberg's work implied that some quantum-mechanical equivalents of classical entities like position and momentum do not commute with one another, as we saw in chapter 17. But Heisenberg himself did not understand that he was using a matrix . It was Heisenberg's mentor Max Born and Born's assistant Pascual Jordan that recognized the matrices.

Independently of Born and Jordan, Dirac saw the non-commutation property of matrices implicit in Heisenberg"s work. He made it the central concept in his mathematical formulation of quantum physics. He called non-commuting quantities q-numbers (for "quantum" or "queer" numbers) and called regular numbers c-numbers (for "classical" or "commuting" numbers).

Dirac grounded his quantum mechanics on three basic ideas, the *principle of superposition*, the *axiom of measurement*, and the *projection postulate*, all of which have produced strong disagreements about the interpretations of quantum mechanics.

But there is complete agreement today that Dirac's theory is the standard tool for quantum-mechanical calculations.

In 1931, Albert Einstein agreed,

> Dirac, to whom, in my opinion, we owe the most perfect exposition, logically, of this [quantum] theory, rightly points out that it would probably be difficult, for example, to give a theoretical description of a photon such as would give enough information to enable one to decide whether it will pass a polarizer placed (obliquely) in its way or not. [1]

1 Einstein, 1931, p.270

This is to remind us that Einstein had long accepted the controversial idea that quantum mechanics is a *statistical* theory, despite the claims of some of his colleagues, notably Born, that Einstein's criticisms of quantum mechanics were all intended to restore determinism and eliminate chance and probabilities.

Einstein's reference to photons passing through an oblique polarizer is taken straight from chapter 1 of Dirac's classic 1930 text, *The Principles of Quantum Mechanics*. Dirac uses the passage of a photon through an oblique polarizer to explain his *principle of superposition*, which he says "forms the fundamental new idea of quantum mechanics and the basis of the departure from the classical theory." [2]

Dirac's principle of superposition is very likely the most misunderstood aspect of quantum mechanics, probably because it is the departure from the deterministic classical theory. Many field-theoretic physicists believe that individual quantum systems can be in a superposition (e.g., a particle in two places at the same time, or going through both slits, a cat "both dead and alive.")

This is the source of much of the "quantum nonsense" in today's popular science literature.

Dirac's projection postulate, or collapse of the wave function, is the element of quantum mechanics most often denied by various "interpretations." The sudden discrete and discontinuous "quantum jumps" are considered so non-intuitive that interpreters have replaced them with the most outlandish alternatives.

David Bohm's "pilot-wave" theory (chapter 30) introduces hidden variables moving at speeds faster than light to restore determinism to quantum physics, denying Dirac's projection probabilities.

Hugh Everett's "many-worlds interpretation" (chapter 31) substitutes a "splitting" of the entire universe into two equally large universes, massively violating the most fundamental conservation principles of physics, rather than allow a diagonal photon arriving at a polarizer to "collapse" into a horizontal or vertical state.

Decoherence theorists (chapter 35) simply deny quantum jumps and even the existence of particles!

John Bell's inequality theorem explaining nonlocality and entanglement depends critically on a proper understanding of

2 Dirac, 1930, p.2

Dirac's principles. It is not clear that Bell fully accepts Dirac's work, as we shall see in chapter 32. The experimental tests of Bell's inequality depend on measuring the polarization or spin of two entangled particles.

Dirac gave a most clear description of the interaction of light particles (photons) with polarizers at various angles in the first chapter of his classic text, *The Principles of Quantum Mechanics*.

To explain his fundamental principle of superposition, Dirac considers a photon which is plane-polarized at a certain angle α and then gets resolved into two components at right angles to one another. How do photons in the original state change into photons at the right-angle states. He says

> "This question cannot be answered without the help of an entirely new concept which is quite foreign to classical ideas... The result predicted by quantum mechanics is that sometimes one would find the whole of the energy in one component and the other times one would find the whole in the other component. One would never find part of the energy in one and part in the other. Experiment can never reveal a fraction of a photon." [3]

At this point Dirac explains how many experiments have confirmed the quantum mechanical predictions for the probabilities of being found in the two components.

> If one did the experiment a large number of times, one would find in a fraction $\cos^2\alpha$ of the total number of times that the whole of the energy is in the α-component and in a fraction $\sin^2\alpha$ that the whole of the energy is in the $(\alpha + \pi/2)$-component. One may thus say that a photon has a probability $\cos^2\alpha$ of appearing in the a-component and a probability $\sin^2\alpha$ of appearing in the $(\alpha + \pi/2)$- component. These values for the probabilities lead to the correct classical distribution of energy between the two components when the number of photons in the incident beam is large. [4]

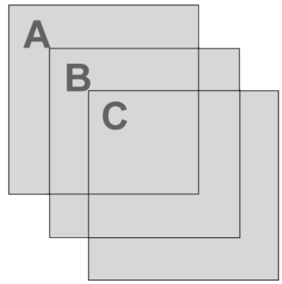
We can illustrate the passage of photons through polarizers turned at different angles, as used in tests of Bell's inequality.

3 ibid., pp.3-4
4 ibid., p.4

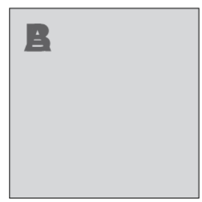
Chapter 19

Dirac's Three Polarizers

We can use three squares of polarizing sheet material to illustrate Dirac's explanation of the quantum superposition of states and the collapse of a mixture of states to a pure state upon measurement or state preparation.



Here are the three polarizing sheets. They are a neutral gray color because they lose half of the light coming though them. The lost light is absorbed by the polarizer, converted to heat, and this accounts for the (Boltzmann) entropy gain required by our new information (Shannon entropy) about the exact polarization state of the transmittted photons.



When polarizers A and B are superimposed we see that the same amount of light comes through two polarizers, as long as the polarizing direction is the same. The first polarizer A prepares the photon in a given state of polarization. The second is then certain to find it in the same state. Let's say the direction of light polarization is vertical when the letters are upright.

If one polarizer, say B, turns 90°, its polarization direction will be horizontal and if it is on top of vertical polarizer A, no light will pass through it.

The Mystery of the Oblique Polarizer

As you would expect, any quantum mechanics experiment must contain an element of "Wow, that's impossible!" or we are not getting to the non-intuitive and unique difference between quantum mechanics and the everyday classical mechanics. So let's look at the amazing aspect of what Dirac is getting to, and then we will see how quantum mechanics explains it.

We turn the third polarizer C so its polarization is along the 45° diagonal. Dirac tells us that the wave function of light passing through this polarizer can be regarded as in a mixed state, a superposition of vertical and horizontal states.

As Einstein said, the information as to the exact state in which the photon will be found following a measurement does not exist.

We can make a measurement that detects vertically polarized photons by holding up the vertical polarizer A in front of the oblique polarizer C. Either a photon comes through A or it does not. Similarly, we can hold up the horizontal polarizer B in front of C. If we see a photon, it is horizontally polarized.

If our measuring apparatus (polarizer B) is measuring for horizontally polarized photons, the probability of detecting a photon diagonally polarized by C is 1/2. Similarly, if we were to measure for vertically polarized photons, we have the same 50% chance of detecting a photon.

Going back to polarizers A and B crossed at a 90° angle, we know that no light comes through when we cross the polarizers.

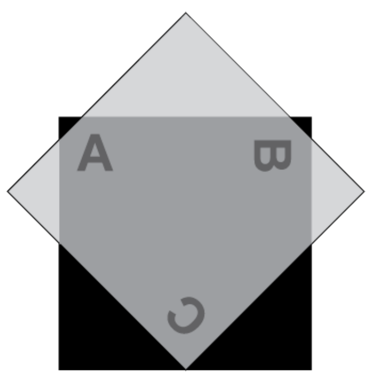
If we hold up polarizer C along the 45° diagonal and place it in front of (or behind) the 90° cross polarizers, nothing changes. No light is getting through.

But here is the amazing, impossible part. If you insert polarizer C at 45° between A and B, some light gets through. Note C is slipped between A (in the rear) and B (in front).

What is happening here quantum mechanically? If A crossed with B blocks all light, how can adding another polarization filter add light?

It is somewhat like the two-slit experiment where adding light by opening a second slit creates null points where light that was seen with one slit open now goes dark.

Here adding another polarizer allows more photons to pass.



Dirac has now introduced the ideas of probability and statistics as a consequence of his principle of superposition. And he now introduces what he calls a "manner of speaking" which is today the source of much confusion interpreting quantum mechanics. He says this way of speaking will help us to "remember the results of experiments," but that "one should not try to give too much meaning to it." Einstein was looking for that deep meaning in reality.

In our polarizing experiment, Dirac suggests that we might speak *as if* a single photon is partly in each of the two states, that it is "distributed" over the two (horizontal and vertical) states.

> When we say that the photon is distributed over two or more given states the description is, of course, only qualitative, but in the mathematical theory it is made exact by the introduction of numbers to specify the distribution, which determine the *weights* with which different states occur in it. [5]

These *weights* are just the probabilities (actually the complex square roots of the probabilities). As Einstein's "objective reality" sees it, an individual photon is always in a single quantum state!

> The description which quantum mechanics allows us to give is merely a manner of speaking which is of value in helping us to deduce and to remember the results of experiments and which never leads to wrong conclusions. One should not try to give too much meaning to it...

Dirac's "manner of speaking" has given the false impression that a single particle can actually be in two states at the same time. This is seriously misleading. Dirac expresses the concern that some would be misled - don't "give too much meaning to it."

But this is something that bothered Einstein for years as he puzzled over "nonlocality." Schrödinger famously used superposition to argue that a cat can be simultaneously dead and alive! (chapter 28).

Many interpretations of quantum mechanics are based on this unfortunate mistake.

> Let us consider now what happens when we determine the energy in one of the components. The result of such a determination must be either the whole photon or nothing at all. Thus the photon must change suddenly from being partly in one beam and partly in the other to being entirely in one of the beams... It is impossible to predict in which of the two beams the photon will be found. Only the probability of either result can be calculated from the previous distribution of the photon over the two beams. [6]

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5 ibid., p.5
6 ibid., p.6

> One cannot picture in detail a photon being partly in each of two states; still less can one see how this can be equivalent to its being partly in each of two other different states or wholly in a single state. We must, however, get used to the new relationships between the states which are implied by this *manner of speaking* and must build up a consistent mathematical theory governing them.[7] [our italics]

Objective Reality and Dirac's "Manner of Speaking"

Dirac's "transformation theory" allows us to "represent" the initial wave function (before an interaction) in terms of a "basis set" of "eigenfunctions" appropriate for the possible quantum states of our measuring instruments that will describe the interaction.

But we shall find that assuming an individual quantum system is actually in one of the possible eigenstates of a system greatly simplifies understanding two-particle entanglement (chapter 29).

This is also consistent with Einstein's objectively real view that a particle has a position, a continuous path, and various properties that are conserved as long as the particle suffers no interaction that could change any of those properties.

Einstein was right when he said that the wave function describes ensembles, that is, the statistical results for large numbers of systems.

All of quantum mechanics rests on the Schrōdinger equation of motion that *deterministically* describes the time evolution of the *probabilistic* wave function, plus Dirac's three basic assumptions, the principle of superposition (of wave functions), the axiom of measurement (of expectation values for observables), and the projection postulate (the "collapse" of the wave function that introduces indeterminism or chance during interactions).

The most appropriate basis set is one in which the eigenfunction-eigenvalue pairs match up with the natural states of the measurement apparatus. In the case of polarizers, one basis is the two states of horizontal and vertical polarization.

Elements in the "transformation matrix" give us the probabilities of measuring the system and finding it in one of the possible quantum states or "eigenstates," each eigenstate corresponding to an "eigenvalue" for a dynamical operator like the energy, momentum, angular momentum, spin, polarization, etc.

7 Dirac, 1930, p.5

Diagonal (n, n) elements in the transformation matrix give us the eigenvalues for observables in quantum state n. Off-diagonal (n, m) matrix elements give us transition probabilities between quantum states n and m.

Notice the sequence - possibilities > probabilities > actuality: the wave function gives us the possibilities, for which we can calculate probabilities. Each experiment gives us one actuality. A very large number of identical experiments confirms our probabilistic predictions. Confirmations are always only statistics, of course.

For completeness, we offer a brief review of the fundamental principles of quantum mechanics, as developed by Paul Dirac.

The Schrōdinger Equation.

The fundamental equation of motion in quantum mechanics is Erwin Schrōdinger's famous wave equation that describes the evolution in time of his wave function ψ.

$$i\hbar\, \delta\psi / \delta t = H\, \psi \qquad (1)$$

Max Born interpreted the square of the absolute value of Schrōdinger's wave function $|\psi_n|^2$ (or $< \psi_n \mid \psi_n >$ in Dirac notation) as providing the probability of finding a quantum system in a particular state n. This of course was Einstein's view for many years.

As long as this absolute value (in Dirac bra-ket notation) is finite,

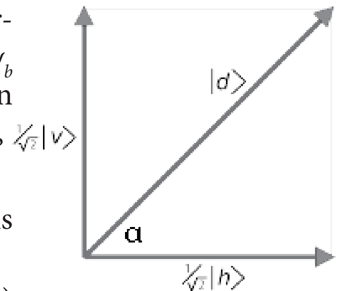
$$< \psi_n \mid \psi_n > = \int \psi^* (q)\, \psi\, (q)\, dq < \infty, \qquad (2)$$

then ψ can be normalized to unity, so that the *probability* of finding a particle somewhere $< \psi \mid \psi > = 1$, which is necessary for its interpretation as a probability. The normalized wave function can then be used to calculate "observables" like the energy, momentum, etc. For example, the probable or expectation value for the position r of the system, in configuration space q, is

$$< \psi \mid r \mid \psi > = \int \psi^* (q)\, r\, \psi\, (q)\, dq. \qquad (3)$$

Dirac's Principle of Superposition.

The Schrōdinger equation (1) is a linear equation. It has no quadratic or higher power terms, and this introduces a profound - and for many scientists and philosophers the most disturbing - feature of quantum mechanics, one that is impossible in classical



physics, namely the principle of superposition of quantum states. If ψ_a and ψ_b are both solutions of equation (1), then an arbitrary linear combination of these,

$$| \psi > = c_a | \psi_a > + c_b | \psi_b >, \qquad (4)$$

with complex coefficients c_a and c_b, is also a solution.

Together with statistical (probablistic) interpretation of the wave function, the principle of superposition accounts for the major mysteries of quantum theory, some of which we hope to resolve, or at least reduce, with an objective (observer-independent) explanation of irreversible information creation during quantum processes.

Observable information is critically necessary for measurements, though we note that observers can come along anytime after new information has been irreversibly recorded in the measuring apparatus as a consequence of the interaction with the quantum system. It is not the "conscious observer" standing by the apparatus that is responsible for the new information coming into existence.

The quantum (discrete) nature of physical systems results from there generally being a large number of solutions ψ_n (called eigenfunctions) of equation (1) in its time independent form, with energy eigenvalues E_n.

$$H \psi_n = E_n \psi_n, \qquad (5)$$

The discrete spectrum energy eigenvalues E_n limit interactions (for example, with photons) to specific energy differences $E_m - E_n$.

In the old quantum theory, Bohr postulated that electrons in atoms would be in "stationary states" of energy E_n, and that energy differences would be of the form $E_m - E_n = h\nu$, where ν is the frequency of the observed spectral line when an atom jumps from energy level E_m to E_n .

Einstein, in 1916, derived these two Bohr postulates from basic physical principles in his paper on the emission and absorption processes of atoms. What for Bohr were postulates or assumptions, Einstein grounded in quantum physics, though virtually no one appreciated his foundational work at the time, and few appreciate it today, his work mostly eclipsed by the Copenhagen physicists.

The eigenfunctions ψ_n are orthogonal to each other

$< \psi_n \mid \psi_m > = \delta_{nm}$ (6)

where the "delta function"

$\delta_{nm} = 1$, if $n = m$, and $= 0$, if $n \neq m$. (7)

Once they are normalized, the ψ_n form an orthonormal set of functions (or vectors) which can serve as a basis for the expansion of an arbitrary wave function φ

$\mid \varphi > = \Sigma_0^\infty c_n \mid \psi_n >$. (8)

The expansion coefficients are

$c_n = < \psi_n \mid \varphi >$. (9)

In the abstract Hilbert space, $< \psi_n \mid \varphi >$ is the "projection" of the vector φ onto the orthogonal axes of the ψ_n "basis" vector set.

Dirac's Axiom of Measurement.

The axiom of measurement depends on Heisenberg's idea of "observables," physical quantities that can be measured in experiments. A physical observable is represented as an operator, e.g., A, that is "Hermitean" (one that is "self-adjoint" - equal to its complex conjugate, $A^* = A$).

The diagonal n, n elements of the operator's matrix,

$< \psi_n \mid A \mid \psi_n > = \int\int \psi^* (q)\, A\, (q)\, \psi\, (q)\, dq$, (11)

are interpreted as giving the (probable) expectation value for A_n (when we make a measurement).

The off-diagonal n, m elements describe the uniquely quantum property of interference between wave functions and provide a measure of the probabilities for transitions between states n and m.

It is the intrinsic quantum probabilities that provide the ultimate source of indeterminism, and consequently of irreducible *irreversibility*, as we shall see.

Transitions between states are irreducibly random, like the decay of a radioactive nucleus (discovered by Rutherford in 1901) or the emission of a photon by an electron transitioning to a lower energy level in an atom (explained by Einstein in 1916).

The axiom of measurement is Dirac's formalization of Bohr's 1913 postulate that atomic electrons will be found in stationary states with energies E_n. In 1913, Bohr visualized them as orbiting the nucleus. Later, he said they could not be visualized, but chemists routinely visualize them as clouds of probability amplitude with easily calculated shapes that correctly predict chemical bonding.

The off-diagonal transition probabilities are the formalism of Bohr's "quantum jumps" between his stationary states, emitting or absorbing energy $h\nu = E_m - E_n$. Einstein explained clearly in 1916 that the jumps are accompanied by his discrete light quanta (photons), but Bohr continued to insist that the radiation was a classical continuous wave for another ten years, deliberately ignoring Einstein's foundational efforts in what Bohr might have felt was his own area of expertise (quantum mechanics).

The axiom of measurement asserts that a large number of measurements of the observable *A*, known to have eigenvalues *An,* will result in the number of measurements with value *An,* that is proportional to the probability of finding the system in eigenstate ψ_n. It is a statistical result that is incomplete, according to Einstein, because it contains only statistical information about an individual measurement. Quantum mechanics gives us only probabilities for finding individual systems in specific eigenstates.

Dirac's Projection Postulate.

Dirac's third novel concept of quantum theory is often considered the most radical. It has certainly produced some of the most radical ideas ever to appear in physics, in attempts by various "interpretations" of quantum mechanics to deny the "collapse of the wave function."

Dirac's projection postulate is actually very simple, and arguably intuitive as well. It says that when a measurement is made, the system of interest will be found in (will instantly "collapse" into) one of the possible eigenstates of the measured observable.

Now the proper choice of the "basis set" of eigenfunctions depends on the measurement apparatus. The natural basis set of vectors is usually one whose eigenvalues are the observables of our measurement system.

In Dirac's bra and ket notation, the orthogonal basis vectors in our example are $| v >$, the photon in a vertically polarized state, and $| h >$, the photon in a horizontally polarized state. These two states are eigenstates of our polarization measuring apparatus.

Given a quantum system in an initial state $|\varphi>$, according to equation 8, we can expand it in a linear combination of the eigenstates of our measurement apparatus, the $|\psi_n>$.

$| \varphi > = \Sigma_0^\infty c_n | \psi_n >.$

In the case of Dirac's polarized photons, the diagonal state $|d>$ is a linear combination of the horizontal and vertical states of the measurement apparatus, $|v>$ and $|h>$.

$|d> = (1/\surd 2) |v> + (1/\surd 2) |h>.$ (12)

When we square the $(1/\surd 2)$ coefficients, we see there is a 50% chance of measuring the photon as either horizontal or vertically polarized.

According to Dirac's axiom of measurement, one of these possibilities is simply made actual, and it does so, said Max Born, in proportion to the absolute square of the complex probability amplitude wave function $|\psi_n|^2$.

In this way, ontological chance enters physics, and it is partly this fact of quantum randomness and indeterminism that bothered both Einstein ("God does not play dice") and Schrōdinger (whose equation of motion for the wave function is deterministic).

But Dirac pointed out that not every measurement is indeterministic. Some measurements do not change the state.

When a photon is prepared in a vertically polarized state $|v>$, its interaction with a vertical polarizer is easy to visualize. We can picture the state vector of the whole photon simply passing through the polarizer unchanged (Pauli's measurement of the first kind).

The same is true of a photon prepared in a horizontally polarized state $|h>$ going through a horizontal polarizer. And the interaction of a horizontal photon with a vertical polarizer is easy to understand. The vertical polarizer will absorb the horizontal photon completely.

Pauli's Two Kinds of Measurement

In the case of a photon simply passing through a polarizer, no new information enters the universe. Wolfgang Pauli called this a *measurement of the first kind*. Measuring a system that is known to be in a given quantum state may only confirm that it is in that state.

Today this is known as a *non-destructive* measurement.

> The method of measurement of the energy of the system discussed till now has the property that a repetition of measurement gives the same value for the quantity measured as in the first measurement...We shall call such measurements the *measurements of the first kind*. On the other hand it can also happen that the system is changed but in a controllable fashion by the measurement - even when, in the state before the measurement, the quantity measured had with certainty a definite value. In this method, the result of a repeated measurement is not the same as that of the first measurement. But still it may be that from the result of this measurement, an unambiguous conclusion can be drawn regarding the quantity being measured for the concerned system before the measurement. Such measurements, we call the measurements *of the second kind*.[8]

Measurements of the second kind are also known as a "state preparation." For example, we can take light of unknown polarization and pass it through a vertical polarizer. Any photon coming through has been *prepared in the vertical state*. All knowledge of the state before such a measurement is lost.

The new information created in a state preparation must be *irreversibly* recorded in the measurement apparatus, in order for there to be something the experimenter can observe. The recording increases the local negative entropy (information), so the apparatus most raise the global entropy, e.g., dissipating the heat generated in making the recording.

The diagonally polarized photon |*d*>, fully reveals the non-intuitive nature of quantum physics. We can visualize quantum indeterminacy, its statistical nature, and we can dramatically

8 Pauli, 1980, p.75

visualize the process of collapse, as a state vector aligned in one direction must rotate instantaneously into another vector direction.

As we saw above, the vector projection of $|d>$ onto $|v>$, with length $(1/\sqrt{2})$, when squared, gives us the probability 1/2 for photons to emerge from the vertical polarizer. But this is only a statistical statement about the expected probability for large numbers of identically prepared photons.

When we have only one photon at a time, we never get one-half of a photon coming through the polarizer. Critics of standard quantum theory, including Einstein, sometimes say that it tells us nothing about individual particles, only ensembles of identical experiments. There is truth in this, but nothing stops us from imagining the strange process of a single diagonally polarized photon interacting with the vertical polarizer.

There are two possibilities. We either get a whole photon coming through (which means that it "collapsed" into a vertical photon, or the diagonal vector was "reduced to" a vertical vector) or we get no photon at all. This is the entire meaning of "collapse." It is the same as an atom "jumping" discontinuously and suddenly from one energy level to another. It is the same as the photon in a two-slit experiment suddenly appearing at one spot on the photographic plate, where an instant earlier it might have appeared anywhere.

We can even visualize what happens when no photon appears. We can say that the diagonal photon was reduced to a horizontally polarized photon and was therefore completely absorbed.

How do we see the statistical nature and the indeterminacy?

First, statistically, in the case of many identical photons, we can say that half will pass through and half will be absorbed.

Secondly, the indeterminacy is simply that in the case of one photon, we have no ability to know which it will be. This is just as we cannot predict the time when a radioactive nucleus will decay, or the time and direction of an atom emitting a photon, as Einstein discovered in 1917, when we first learned that ontological chance is involved in quantum processes, especially in the interaction of matter and radiation.

This indeterminacy is a consequence of our diagonal photon state vector being "represented" (transformed) into a linear superposition of vertical and horizontal photon basis state vectors.

It is the principle of superposition together with the projection postulate that provides us with indeterminacy, statistics, and a way to "visualize" the collapse of a superposition of quantum states into one of the basis states.

Quantum mechanics is a probabilistic and statistical theory. The probabilities are theories about what experiments will show.

Theories are confirmed (statistically) when a very large number of experiments are performed with identical starting conditions.

Experiments provide the statistics (the frequency of outcomes) that confirm the predictions of quantum theory - with the highest accuracy of any physical theory ever invented!

But Dirac's principle of superposition of states, which gives us the probabilities of a system being found in different eigenstates, never means an *individual* system is in a combination of states!

Schrödinger's Cat (chapter 28) is always found to be dead or alive, not some bizarre combination of both.

And as Dirac made perfectly clear, we never find a photon split between a partial photon vertically polarized and another part horizontally polarized.

We always find the whole photon (or electron). And there is no reason that before the measurement, the particle is in some combination or superposition of states and lacks properties such as position, momentum, angular momentum, all of which are conserved quantities according to their conservation laws.

Thus Einstein's view of "objective reality," that particles have paths between measurements, is in complete agreement with Dirac's transformation theory.

We shall see in chapter 24, that the Copenhagen Interpretation denies Einstein's very simple and intuitive views of "reality."