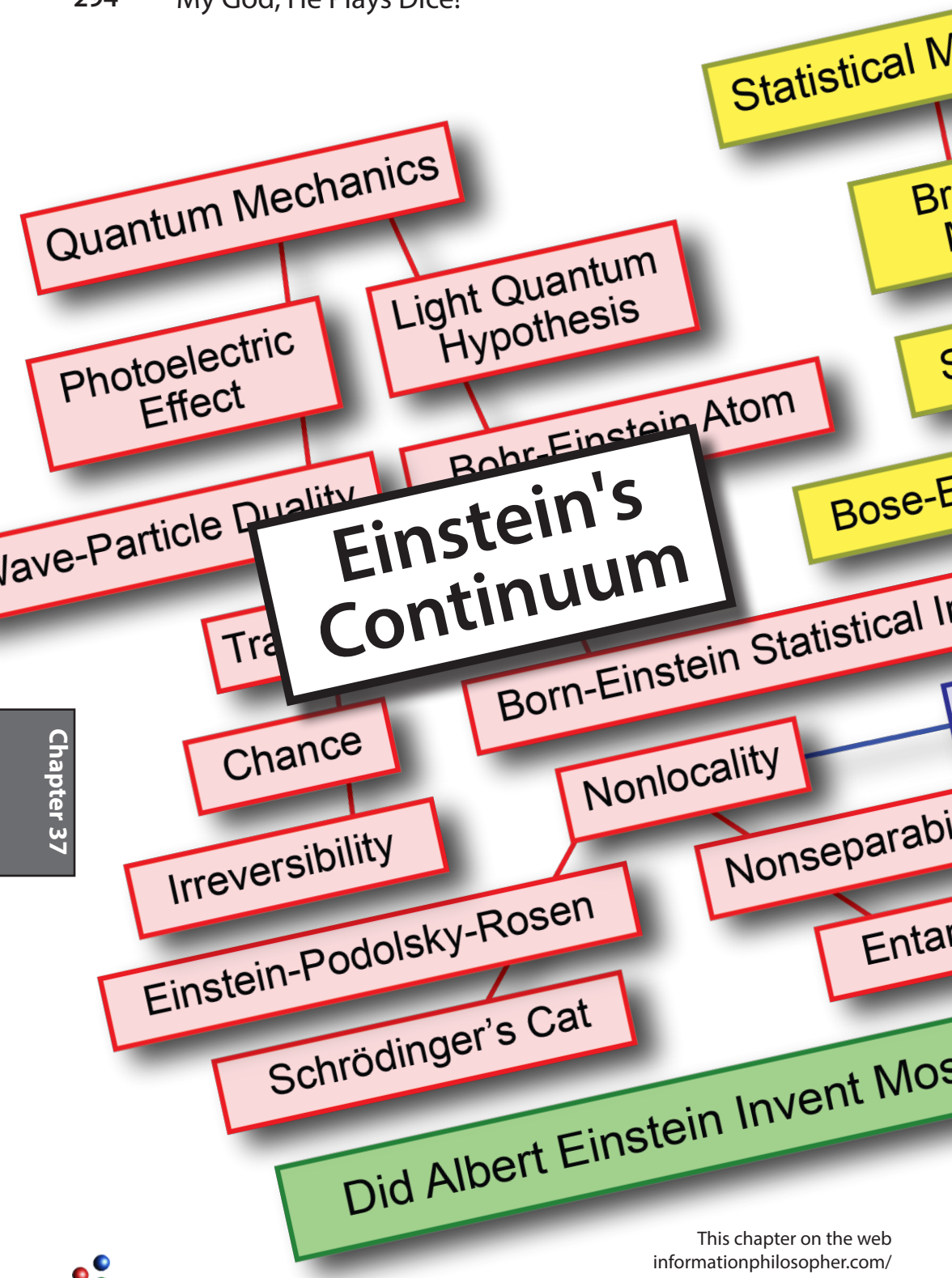

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Einstein's Continuum

Is the Nature of Reality Continuous or Discrete?

Is it possible that the physical world is made up of nothing but discrete discontinuous *particles*? Are continuous *fields* with well-defined, arbitrarily accurate, values for the field at all places and times simply theoretical constructs, confirmed only by averages over large numbers of particles?

Space and time themselves have well-defined values everywhere, but are these just the abstract *information* of the *ideal* coordinate system that allows us to keep track of the positions and motions of particles? Space and time are physical, but they are not *material*.

We use material things, rulers and clocks, to measure space and time. We use the abstract mathematics of real numbers and assume there are an *infinite number* of real points on any line segment and an infinite number of moments in any time interval. But are these continuous functions of space and time nothing but *immaterial* ideas with no material substance?

The two great physical theories at the end of the nineteenth century, Isaac Newton's classical mechanics and James Clerk Maxwell's electrodynamics, are *continuous field theories*.

Solutions of their field equations determine precisely the exact forces on any material particle, providing complete information about their past and future motions and positions. Field theories are generally regarded as *deterministic* and *certain*.

Although the dynamical laws are "free inventions of the human mind," as Einstein always said,[1] and although they ultimately depend on experimental evidence, which is always *statistical*, the field theories have been considered superior to merely statistical laws. Dynamical laws are thought to be *absolute*, based on *principles*.

1 "Geometry and Experience," in *Ideas and Opinions*, p.234

We will find that the continuous, deterministic, and analytical laws of classical dynamics and electromagnetism, expressible as differential equations, are idealizations that "go beyond experience."

A continuum is approached in the limit of large numbers of particles, when the random *fluctuations* of individual events can be averaged over. But this is like the limit theorems of the differential calculus, when large numbers are allowed to go to infinity, and infinitesimals are assumed to reach zero.

All field theories use *continuous* functions that introduce mathematical infinities and infinitesimals. Einstein suspected these infinities may only "exist" in human minds. He learned this from the great mathematicians Leopold Kronecker and Richard Dedekind.

Einstein discovered his favorite phrase "free creations of the human mind" in the work of Dedekind (*freie Schöpfungen des menschlichen Geistes*) so Einstein also knew very well Dedekind's argument that all the axioms of Euclid's geometry can be proven with no reference to a continuum between geometric points. A discrete algebraic theory would be equally good, said Dedekind.

> If anyone should say that we cannot conceive of space as anything else than continuous, I should venture to doubt it and to call attention to the fact that a far advanced, refined scientific training is demanded in order to perceive clearly the essence of continuity and to comprehend that besides rational quantitative relations, also irrational, and besides algebraic, also transcendental quantitative relations are conceivable. [2]

God Created the Integers

Einstein was assuredly also familiar with Kronecker's famous quote "God has made the integers, all else is the work of man." (*Die ganzen Zahlen hat der liebe Gott gemacht, alles andere ist Menschenwerk*). These ideas must have given Einstein a healthy skepticism about his work on *continuous* field theories. In his later

2 Dedekind, 1901, p.38

years, Einstein gave thought to algebraic or *discrete* difference equations, not continuous differential equations.

Einstein may have even doubted the "existence" of the integers. He and Leopold Infeld wrote in the book, *The Evolution of Physics,*

> Science is not just a collection of laws, a catalogue of unrelated facts. It is a creation of the human mind, with its freely invented ideas and concepts... "Three trees" is something different from "two trees." Again "two trees" is different from "two stones." The concepts of the pure numbers 2, 3, 4..., freed from the objects from which they arose, are creations of the thinking mind which describe the reality of our world. [3]

Experiments that support physical laws are always finite in number. The number of particles in the observable universe is finite. Experimental evidence is always *statistical*. It always contains *errors* distributed randomly around the most probable result, like the fluctuations of entropy around its maximum.

The smooth Gaussian bell curve approached when a very large number of independent random events is plotted is clearly an idealization. That Bell curve is clearly an "idea," a "free creation of the human mind. "

Einstein was gravely concerned that there is nothing in his "objective reality" corresponding to this continuum.

> From the quantum phenomena it appears to follow with certainty that a finite system of finite energy can be completely described by a finite set of numbers (quantum numbers). This does not seem to be in accordance with a continuum theory, and must lead to an attempt to find a purely algebraic theory for the description of reality. [4]

To Leopold Infeld, Einstein wrote in 1941,

> "I tend more and more to the opinion that one cannot come further with a continuum theory." [5]

3 Einstein and Infeld, 1961, p.294
4 Einstein, 1956, p.165
5 Pais, 1982, p.467